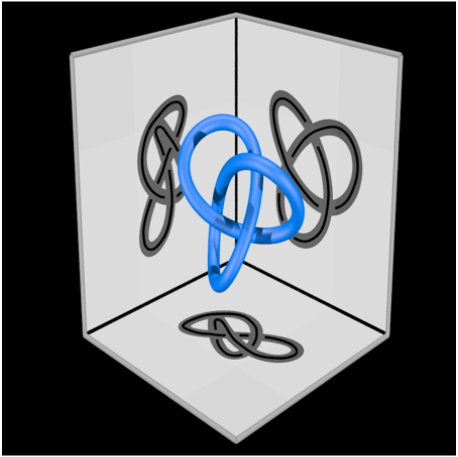
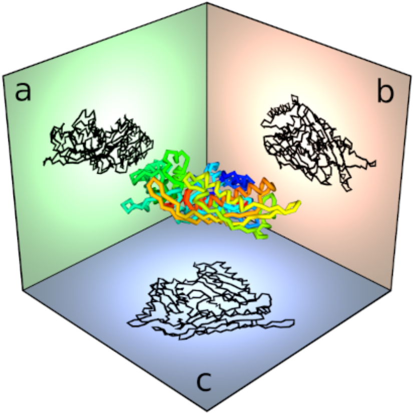
What is...crossing number additivity?

Or: Adding crossings adds crossings?

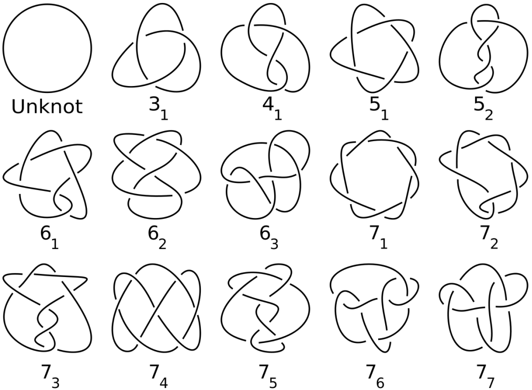
Crossings in projections





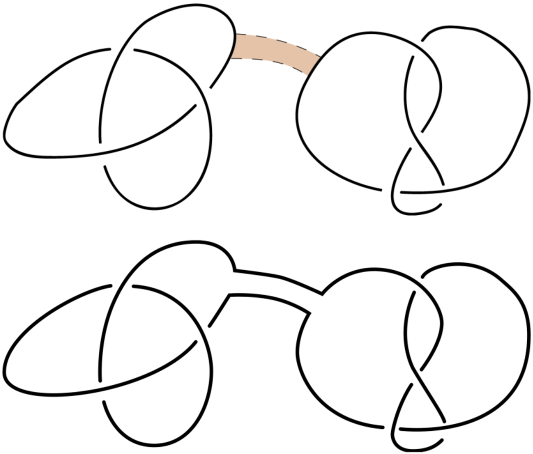
- Knots live in three-space and do not cross
- But their 2d projections=shadows cross
- The number of crossings depends on the projection

The crossing number



- The crossing number $cr(K)$ of a knot K is the minimum number of crossings running over all projections
- Most knot tables are ordered by crossing number
- Problem Computing the crossing number is essentially impossible

Crossing number under addition=connected sum $\#$



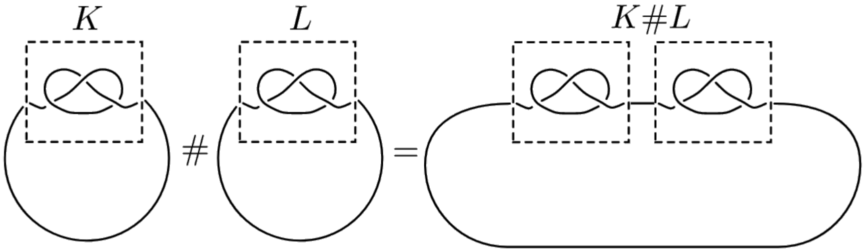
- Question Is $cr(K) + cr(L) = cr(K\#L)$?
- "Adding crossings adds crossings?" is one of the biggest open problems in knot theory

Enter, the theorem

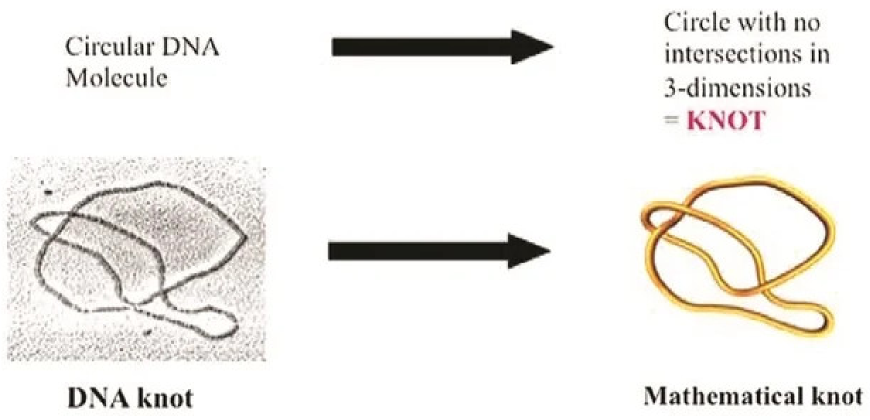
We have

$$\frac{1}{152}\big(cr(K)+cr(L)\big) \leq cr(K\#L) \leq cr(K)+cr(L)$$

- $cr(K\#L) \leq cr(K)+cr(L)$ as $K\#L$ is a projection with $cr(K)+cr(L)$ crossings
- The main statement is the lower bound
- For some knot classes we know equality, *e.g.* for alternating knots



Nature knows this is hard...?



- $cr(\_)$ and the physical behavior of DNA knots are related
- For prime DNA knots $cr(\_)$ is a good predictor of certain behavior
- For composite knots this seems to be wrong

Thank you for your attention!

I hope that was of some help.