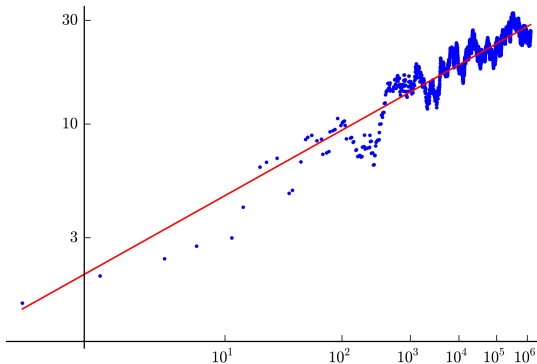


What is...experimental mathematics?

Or: Subfields of mathematics 8

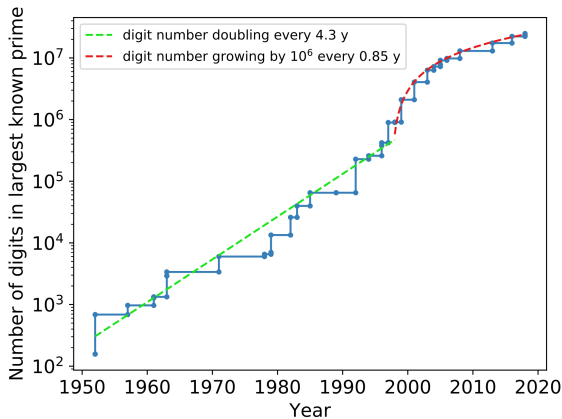
Mathematicians do experiments, too!

The Birch–Swinnerton–Dyer conjecture was discovered using computer experiments



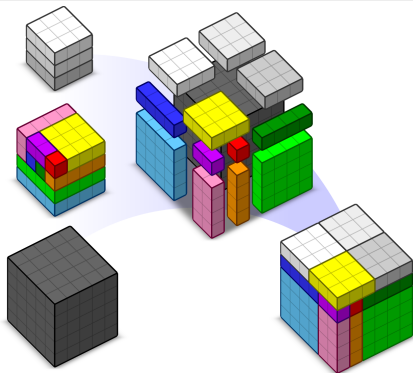
-
- ▶ **Experiments** are everywhere in mathematics – just in disguise
 - ▶ **Paul Halmos quote** Mathematics is not a deductive science—that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork. You want to find out what the facts are, and what you do is in that respect similar to what a laboratory technician does

Searching for examples



- ▶ **Mersenne prime** A prime of the form $2^n - 1$
- ▶ **Problem** Its difficult to decide whether $2^n - 1$ is prime for fixed n
- ▶ **Experiment** Use a computer to find 'large' Mersenne primes

Searching for counterexamples



$$3^3 + 4^3 + 5^3 = 6^3$$

- ▶ Euler's sum of powers conjecture $a_1^k + \dots + a_n^k = b^k$ implies $n \geq k$
- ▶ **Problem** This is a version of Fermat's last theorem, so expected to be difficult
- ▶ **Experiment** Use a computer to find counterexamples such as $27^5 + 84^5 + 110^5 + 133^5 = 144^5$

Enter, the theorem

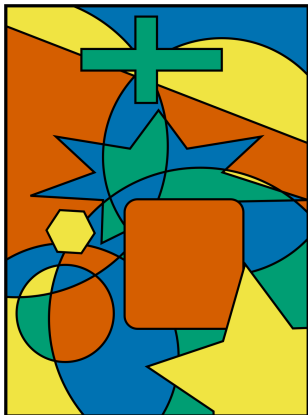
Theorem without words

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$



-
- ▶ The formula was discovered using computer experiments
 - ▶ Honorable mentions Finding patterns (e.g. using visual methods), symbolic validation (e.g. solutions to a special case) etc.
 - ▶ Experimental mathematics answers similar questions!

Also on the list, but that is a bit strange



By Ken Hansen

A student of mine asked me to try to give him a reason for a fact which I did not know was a fact - and so I did not. He says that if a figure is any how divided and the compartments differently colored so that figures with any piece of common boundary have no different colors - four colors may be used - but not more - the following is his case on which four are wanted

A B C D are names of colors



Every correct necessity for 4 for a map to be colored is for as I see at this moment, if four compartments have one boundary line in common with one of the others, then of these sides the fourth, and prevent any path from remaining with it. If this be true, four colors will color any possible map without any necessity for the color meeting color point at a point.

Now it does seem that drawing three compartments with common boundary A B C two and two - you cannot



makes a fourth like boundary from all, making nothing new - that it is tricky work and I am often all conclusions - what do you say? And how, if I have been asked to color a map of England, B is included



The man I think of it the more accident it seems. If you what with me very simple case which makes me out a student would, I think I must be on the highway did of his side. Be true the following proposition of logic follows

If A B C D be four names of which any two might be separated by boundary then some set of definition, then some one of the names must be a name of some name which includes nothing external to the other three

Your truly
W. D. King

7 Oct 27
Oct 25/27

- ▶ Four color theorem Any map can be colored with at most four colors
- ▶ Problem Every known proof contains too many cases to check by hand
- ▶ Experiment Use a computer to do the case-by-case check

Thank you for your attention!

I hope that was of some help.