What is...topological data analysis?

Or: Subfields of mathematics 3

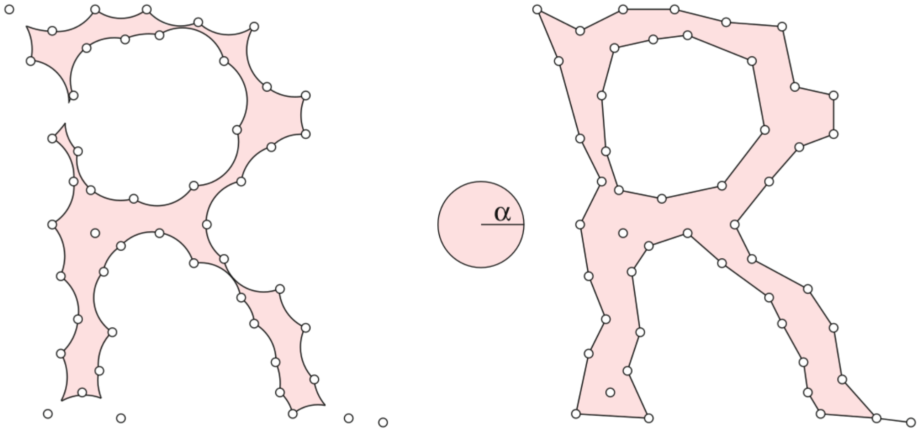
The study of shapes

In topology a cow and a sphere are the same!

$\cong$

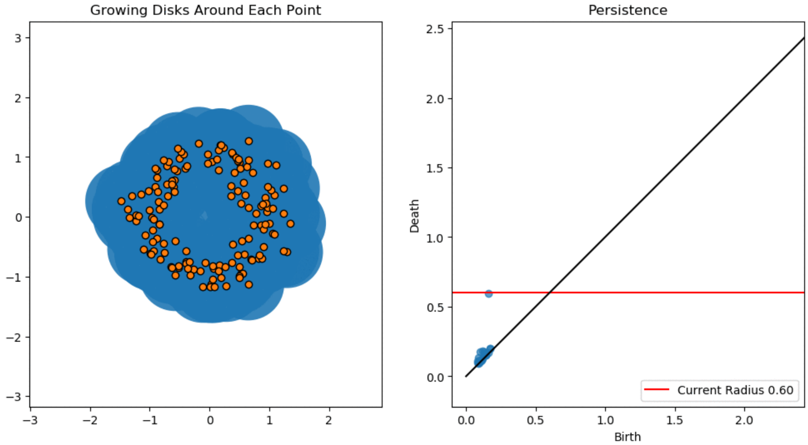
- **Topology** = the study of shape
- **Example** The main stars of topology are continuous maps: topologists never study $x^2 + y^2 = 1$ itself but rather the class of its continuous deformations
- **Question** Is it 'useful' to study shapes?

Enter: topological data analysis (TDA)



- Say we have a point cloud of data and we want to known its 'shape'
- Form discs of radius α; the α hull is the complement of the union of the discs hitting no point; the α shape is obtained by straightening the edges
- TDA provides methods to study the 'shape of data'

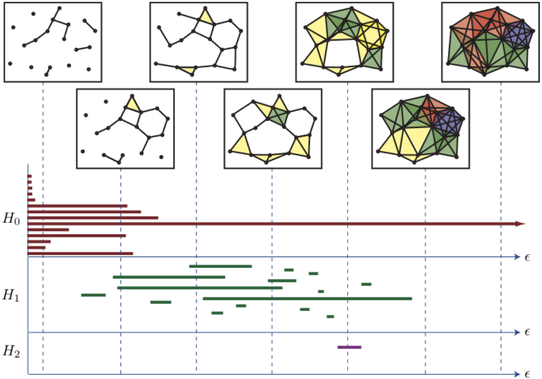
Persistent homology



- Persistent homology = measure the shape of data using growing discs
- Better than an explanation is an animation
- Example The 1th persistent homology measures how internal circles change

Enter, the theorem

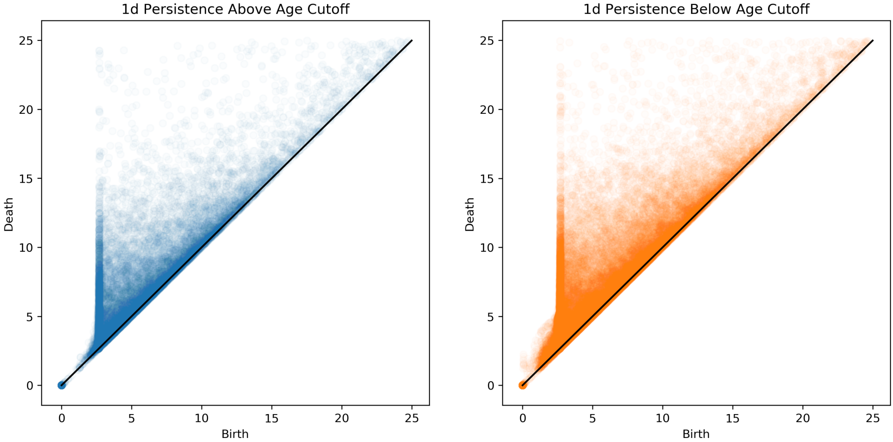
Persistent homology is visualized through a barcode diagram (makes sense because of the theorem below):



Theorem Any finitely generated persistence module has only free and torsion parts

- Free = things that survive; torsion = things that die
- Topological data analysis answers similar questions!

Real-world applications of TDA



- Homology proved useful in detecting age differences in brain artery trees
- Idea Render brain artery trees into point-clouds and use persistent homology
- Differences are subtle – like most differences in human brains – but measurable

Thank you for your attention!

I hope that was of some help.