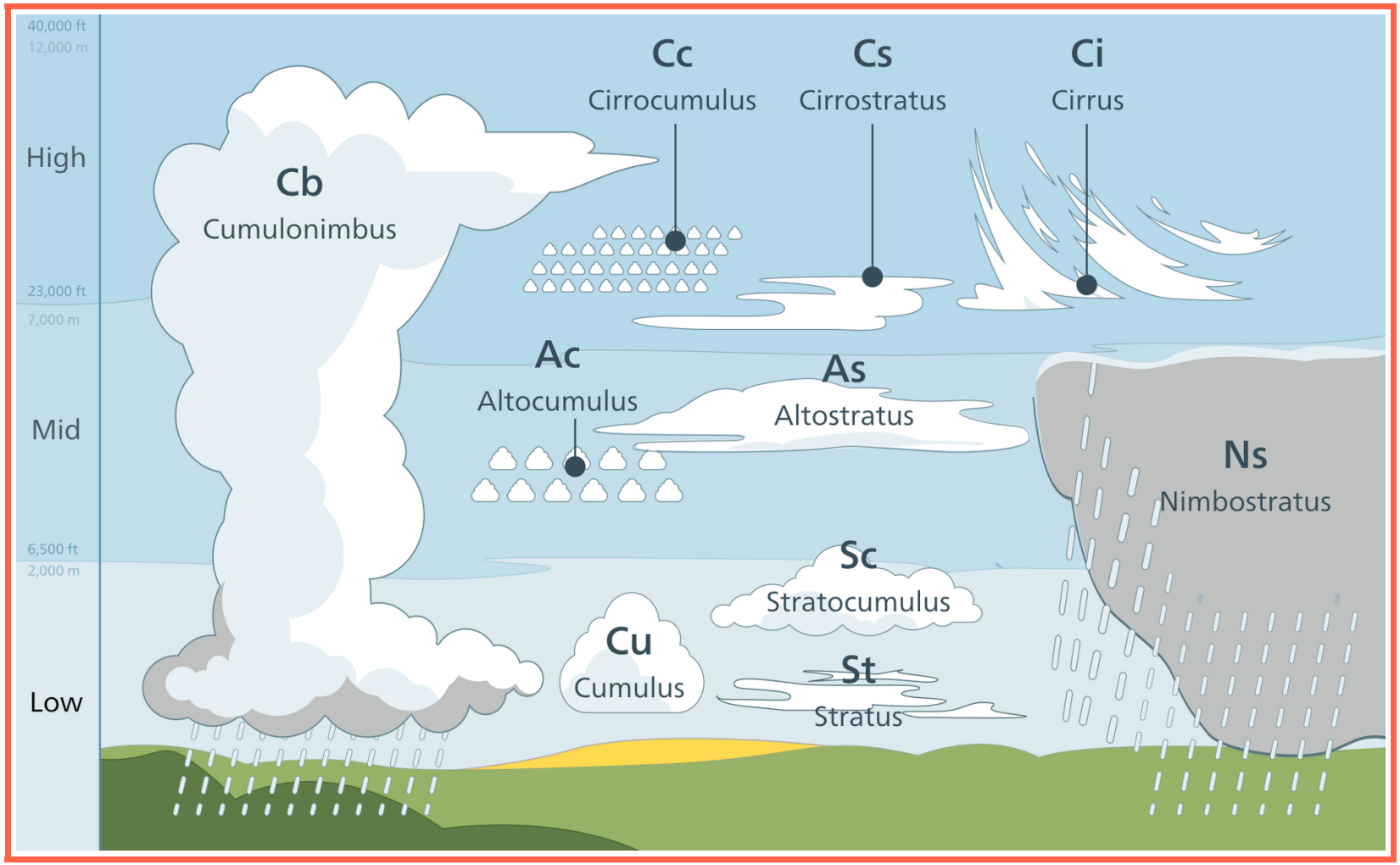
What is...random geometry?

Or: Can chance produce predictable shapes?

Why study random things at all?



- Gases We cannot track everything, yet we can predict pressure and temperature
- Networks Random contacts help model epidemics, news, and computer viruses
- Shapes Cracks, clouds, coastlines: can their irregular patterns follow laws?

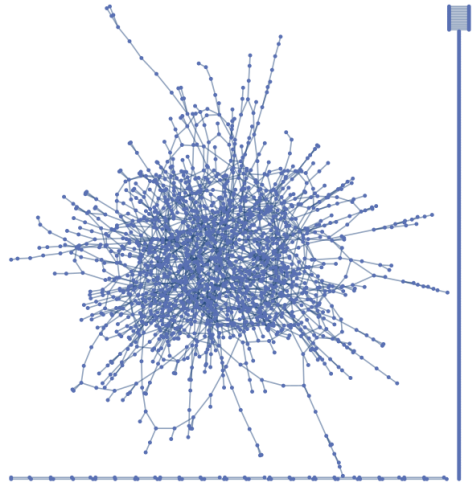
Early examples: make the geometry random

$G_{n,p}$ = random graph with n vertices and probability p to connect two vertices

(i) If p is constant, then $G_{n,p}$ is connected with probability 1 for $n \gg 0$

(ii) If $p = 1/n$, then $G_{n,p}$ has a giant component with probability 1 for $n \gg 0$

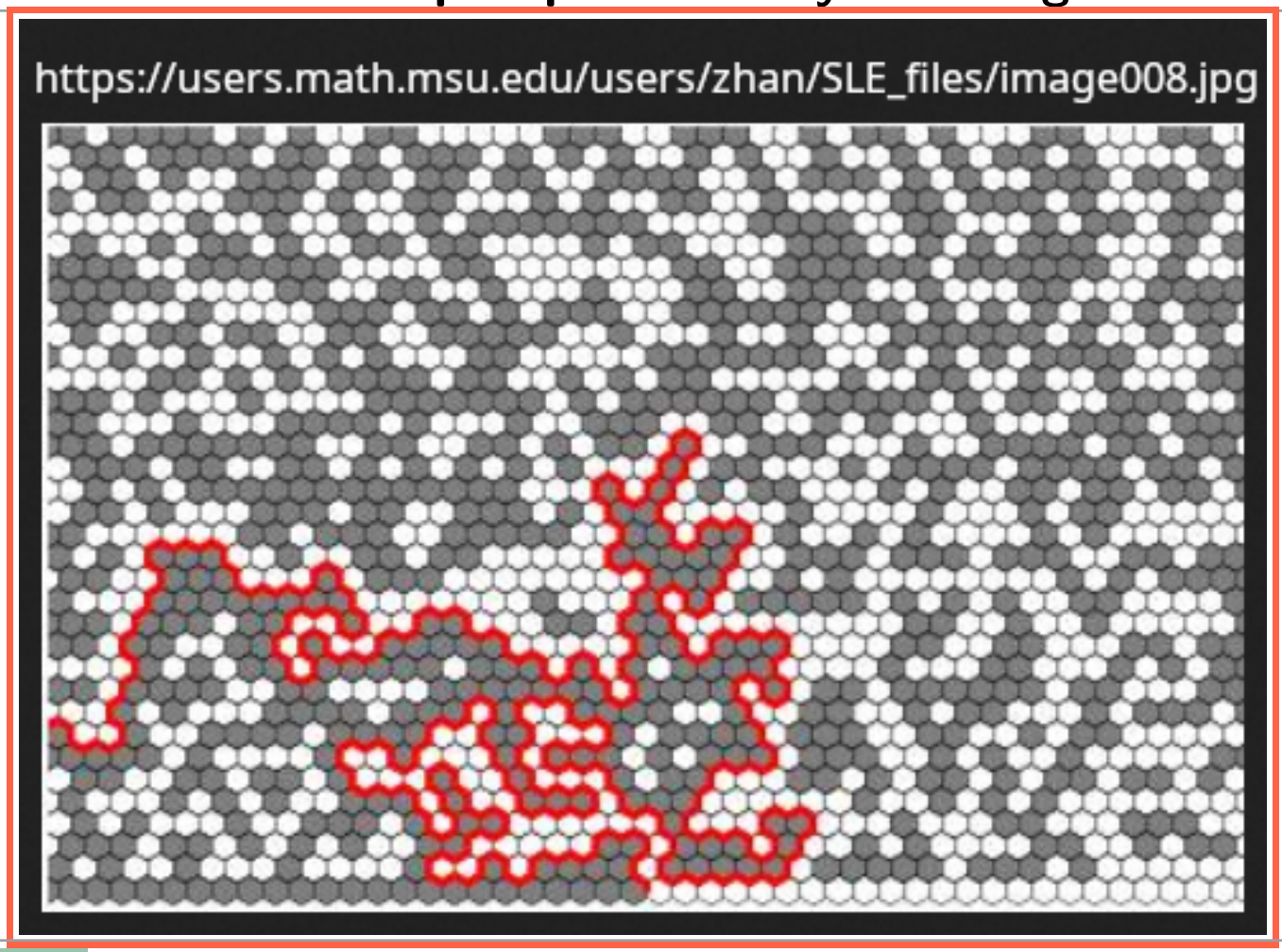
Giant component:



- Better $(p < \frac{1-\epsilon}{n}) \Rightarrow$ a lot of tiny components, $(p = 1/n) \Rightarrow$ giant component, $(p > \frac{1+\epsilon}{n}) \Rightarrow$ connected
- Random graph theory answers similar questions!

- Random geometry The links in a network, or the triangles on a sphere, are chosen by chance (what that means in practice is case dependent)
- Tutte, 1960s Counted planar maps: ways to draw networks on a sphere
- Physics, 1980s Used random maps to model fluctuating 2d surfaces

2000s example: patterns beyond the grid



- Percolation Color hexagons black or white, each with probability $1/2$; trace a random path separating the two colors
- Schramm, 2000 SLE constructs random curves using one-dimensional Brownian motion; a parameter controls how wildly they wind
- 2001–2008 As the hexagons shrink, the path converges to SLE_6, a random curve of fractal dimension $7/4$

2010s example: a random sphere emerges

spring-electrical embedding?

From https://arxiv.org/pdf/1403.7943

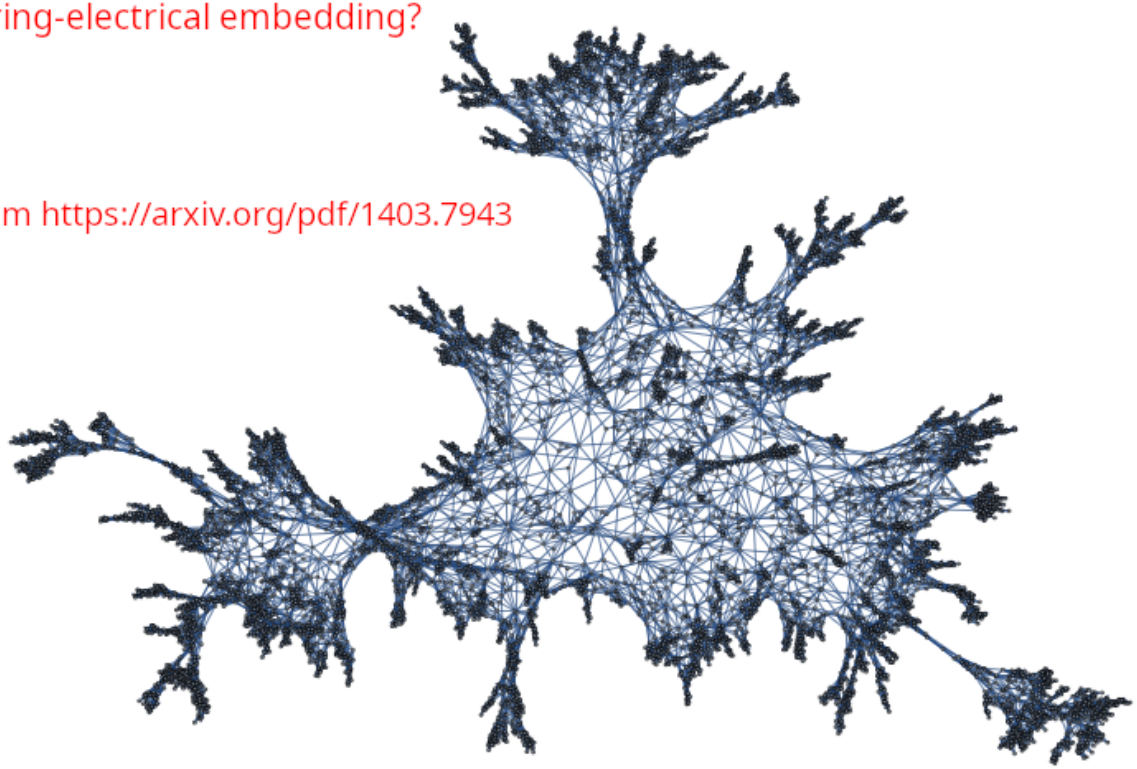
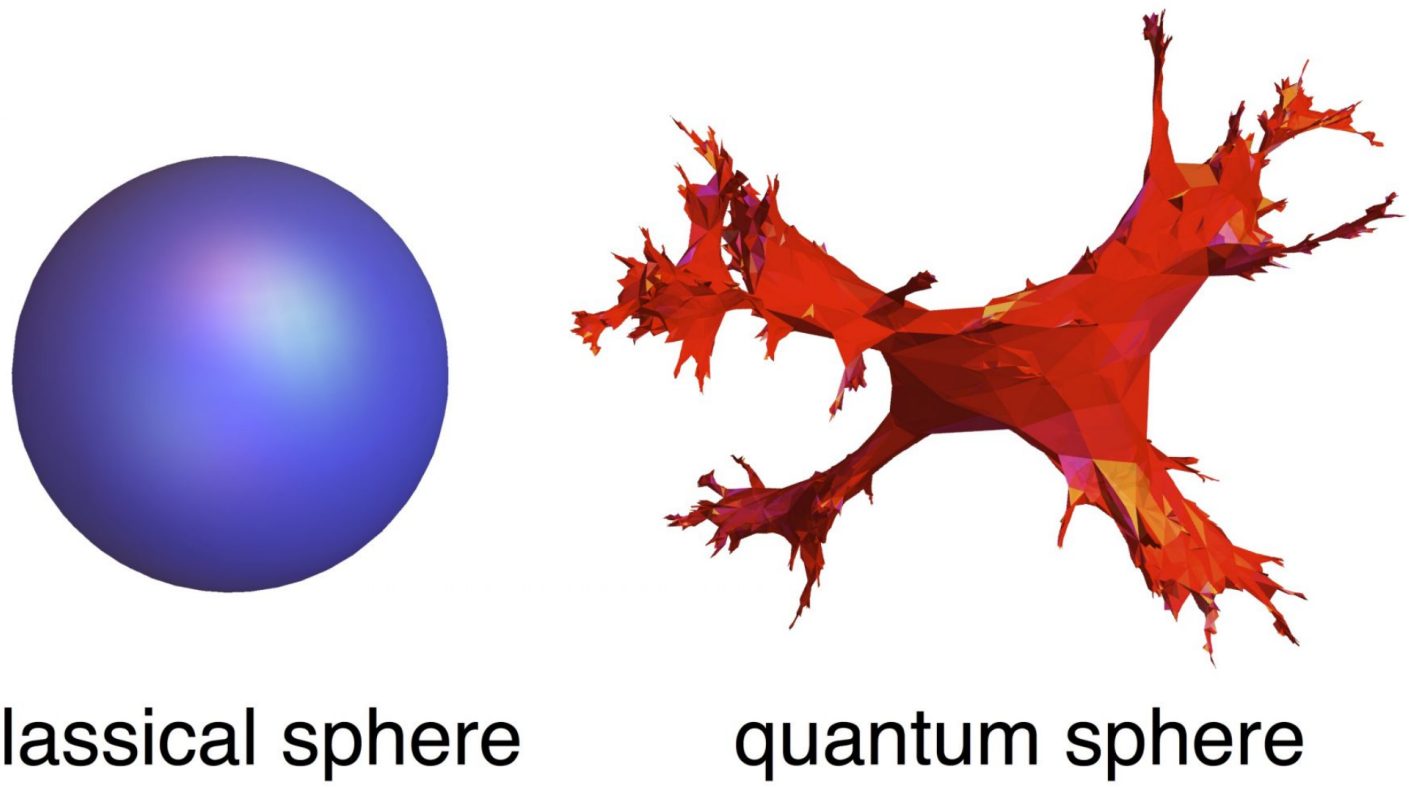


Figure 3. Simulation of a large triangulation of the sphere. Here only the graph structure is represented in three dimensions.

- Random triangulations Divide a sphere into n triangles at random; measure distances by counting edges
- Le Gall, Miermont Typical distances grow like $n^{1/4}$; after rescaling, the maps converge to the Brownian map
- The surprise The limit is topologically a sphere, but has fractal dimension four! Different models give the same limit

2020s example: another way to make a random sphere

From https://www.ipht.fr/wp-content/uploads/2015/02/spheres_3.jpg



- A random landscape Start with an ordinary sphere and imagine a random pattern of hills and valleys, fluctuating at every scale
- Liouville quantum gravity Use this pattern to stretch and shrink the geometry: different regions get different areas and distances
- Miller–Sheffield, 2021 Surprisingly, a particular version produces exactly the same random geometry as the Brownian map!

Thank you for your attention!

Next time: Infinity-categories