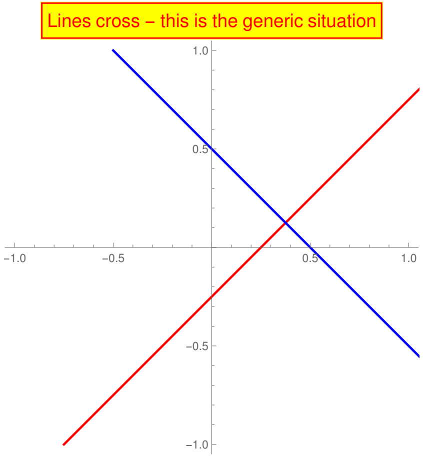
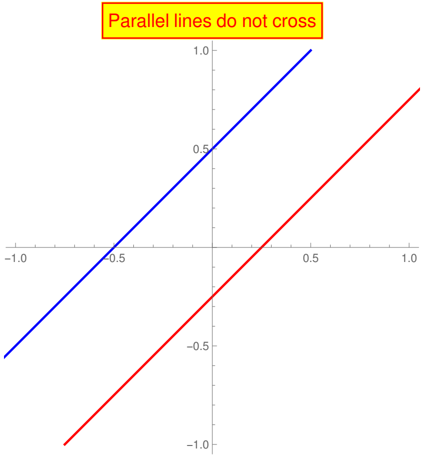
What is...a projective space?

Or: My lines cross.

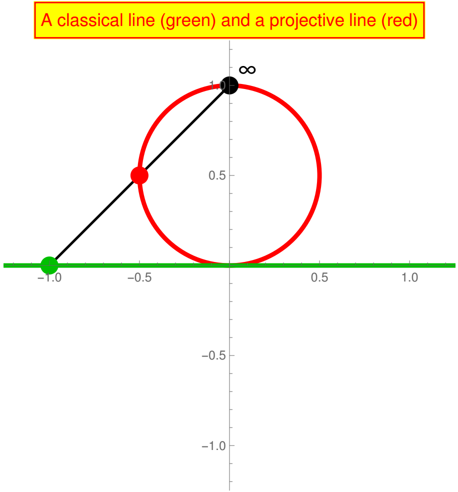
Lines always cross, right?





That parallel lines do not cross is a “problem” of classical/affine geometry

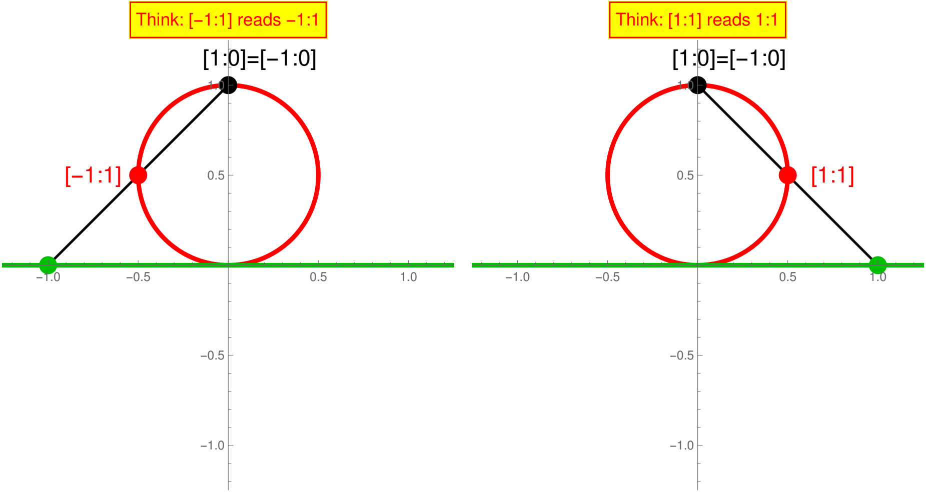
A classical line and a projective line



The green point corresponds to the red point

The real projective line $\mathbb{P}(\mathbb{R}^2)$ includes a point at infinity and a classical line

Projective coordinates



This is not quite a dimension higher – only ∞ needs the second coordinate

For completeness: A formal definition.

For a vector space V over a field $\mathbb{K}$ the projective space $\mathbb{P}(V)$ is the set of equivalence classes of $V \setminus \{0\}$ under $v \sim w \Leftrightarrow v = \lambda w$. If $v = x_0 v_0 + ... + x_n v_n$ in a basis $\{v_0, ..., v_n\}$ of V, then $[x_0 : ... : x_n]$ are the projective coordinates

Projective coordinates: $[x_0 : ... : x_n] = [\lambda x_0 : ... : \lambda x_n]$

(Vector space up to scaling)

Important facts:

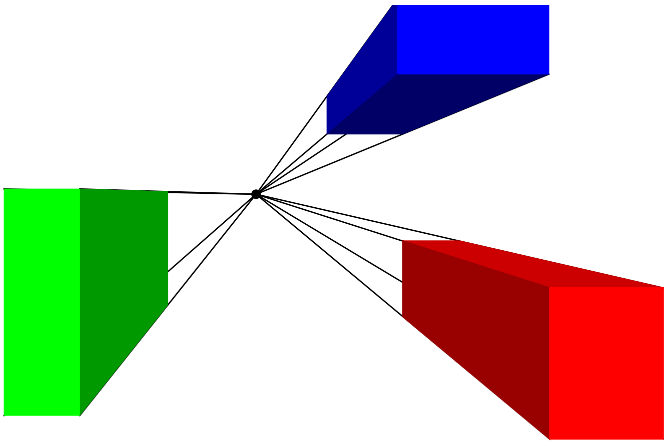
(a) $[x_0 : ... : x_{n-1} : 0]$ are points at infinity

(b) The points $[x_0 : ... : x_{n-1} : x_n \neq 0]$ are classical points via $[x_0/x_n : ... : x_{n-1}/x_n : 1]$

(c) $\mathbb{P}(V)$ has $\dim \mathbb{P}(V) = \dim V - 1$ (Information need to determine points)

(d) Two projective lines in the same plane meet in at least one point

Projective geometry is the geometry of perspective



∞ is the point at the horizon

Thank you for your attention!

I hope that was helpful.