What is...a linear subspace?

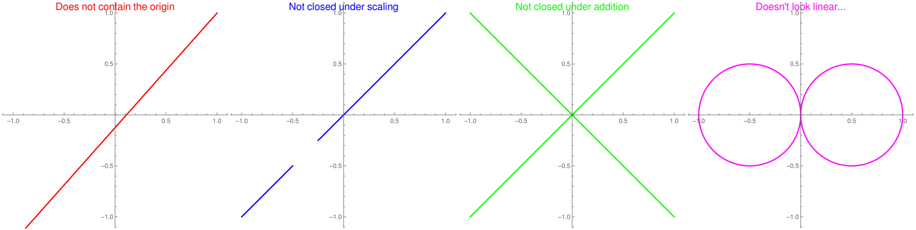
Or: Hyperplanes and friends.

My wish list for a substructure (in mathematics and beyond).

V is some object of type XYZ. A substructure W should be:

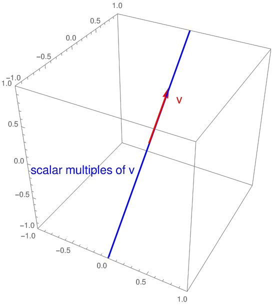
- I want that W is inside of V
- I want that W is not empty (that would be silly...)
- I want that W is also of type XYZ, a.k.a. closed under type XYZ operations

Things we do not want to be substructures of $\mathbb{R}^2$:

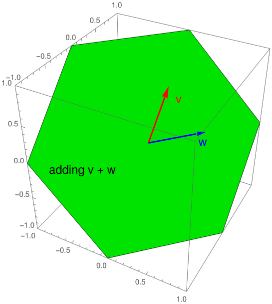


Hyperplanes – closed under linear operations

$(v \in W \Rightarrow \lambda v \in W) \leftrightsquigarrow$



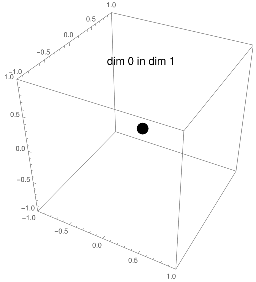
$(v, w \in W \Rightarrow v + w \in W) \leftrightsquigarrow$

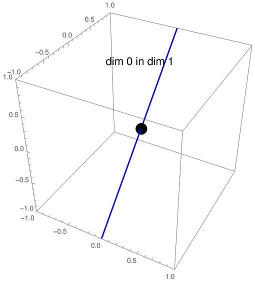


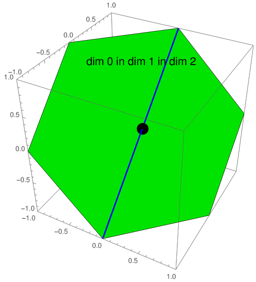
A space in a space in a space in a...

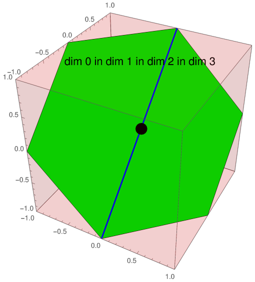
A sequence of linear subspaces of $\mathbb{R}^3$:

$$0 \subset \mathbb{R}\{(0,1,1)\} \subset \mathbb{R}\{(0,1,1),(1,1,0)\} \subset \mathbb{R}^3$$









For completeness: A formal definition

A linear subspace W of a vector space V (over $\mathbb{K}$) satisfies:

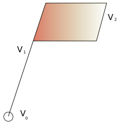
- W is a subset of V
- $W \neq \emptyset$
- For all $\lambda, \mu \in \mathbb{K}$ and all $v, w \in W$ we have $\lambda v + \mu w \in W$

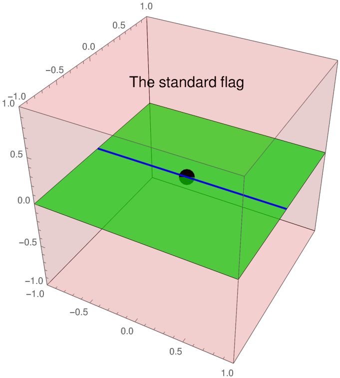
Important facts:

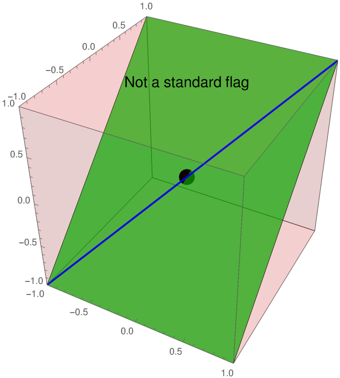
- W is itself a vector space
- All linear maps $f\colon V \to X$ restrict to linear maps $f|_W\colon W \to X$

Flags

A real flag: $$0 = V_0 \subset V_1 \subset \ldots \subset V_{n-1} \subset V_n = \mathbb{R}^n \qquad \dim V_i = i$$ $\leftrightsquigarrow$







Thank you for your attention!

I hope that was of some help.