Lie theory - part 31

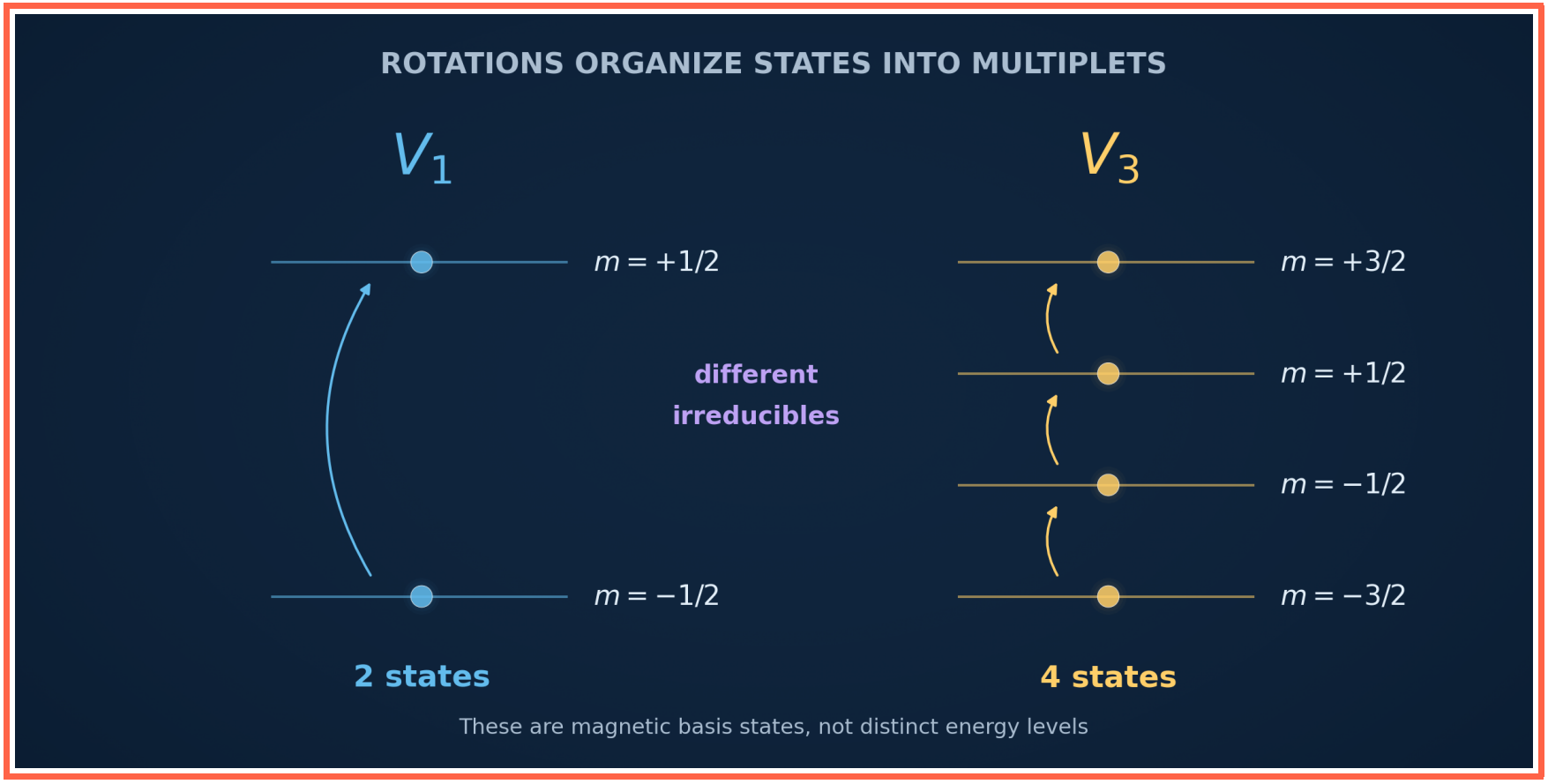
Or: Why atoms have favourite colours

Why do sodium lamps look so yellow?



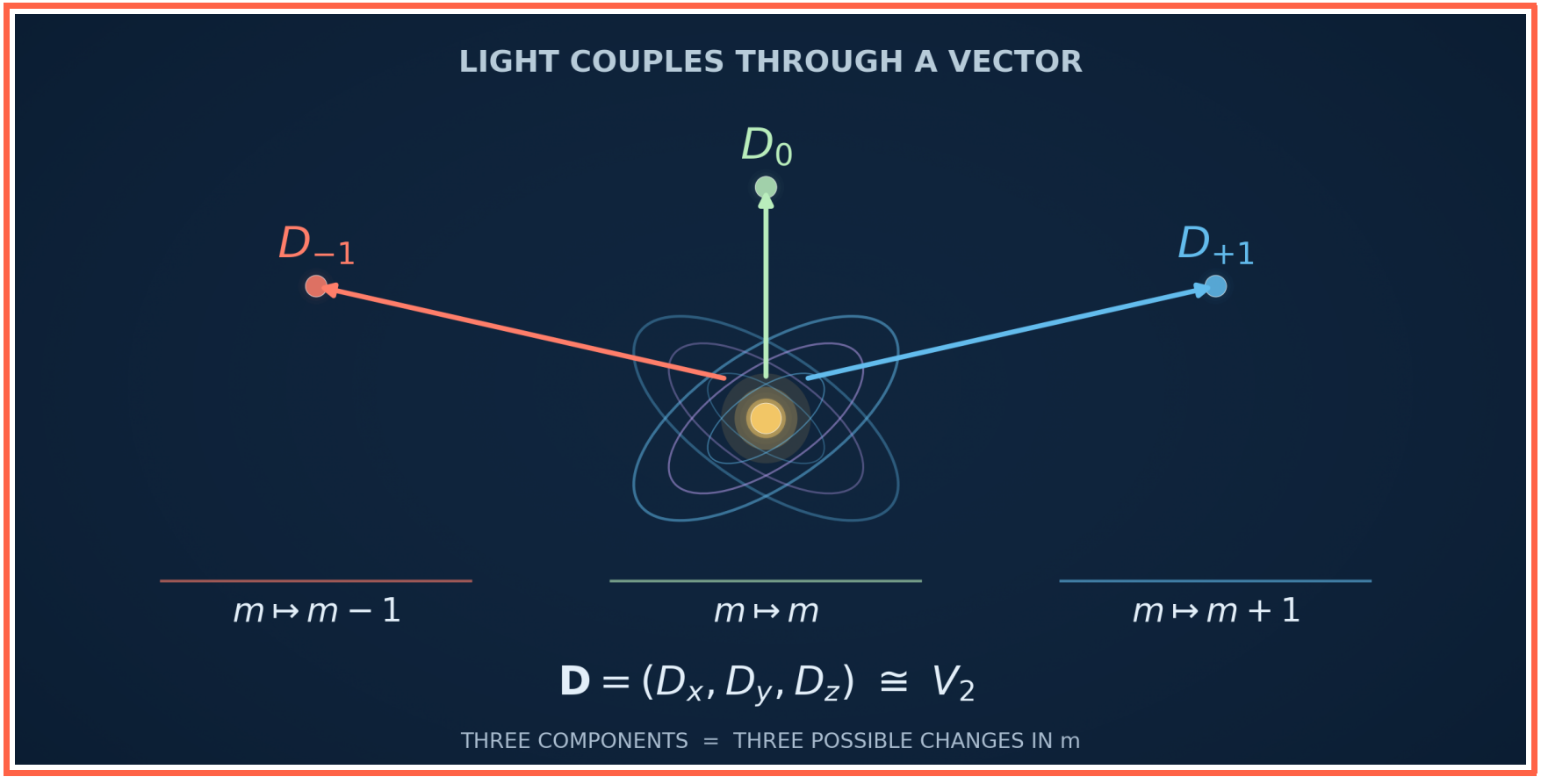
- The picture Some lamps glow in a remarkably narrow band of colours.
- The mechanism An atom releases light when it drops between energy levels.
- The puzzle Why are some imaginable jumps absent or very faint?

The $SU(2)$ inside an atom



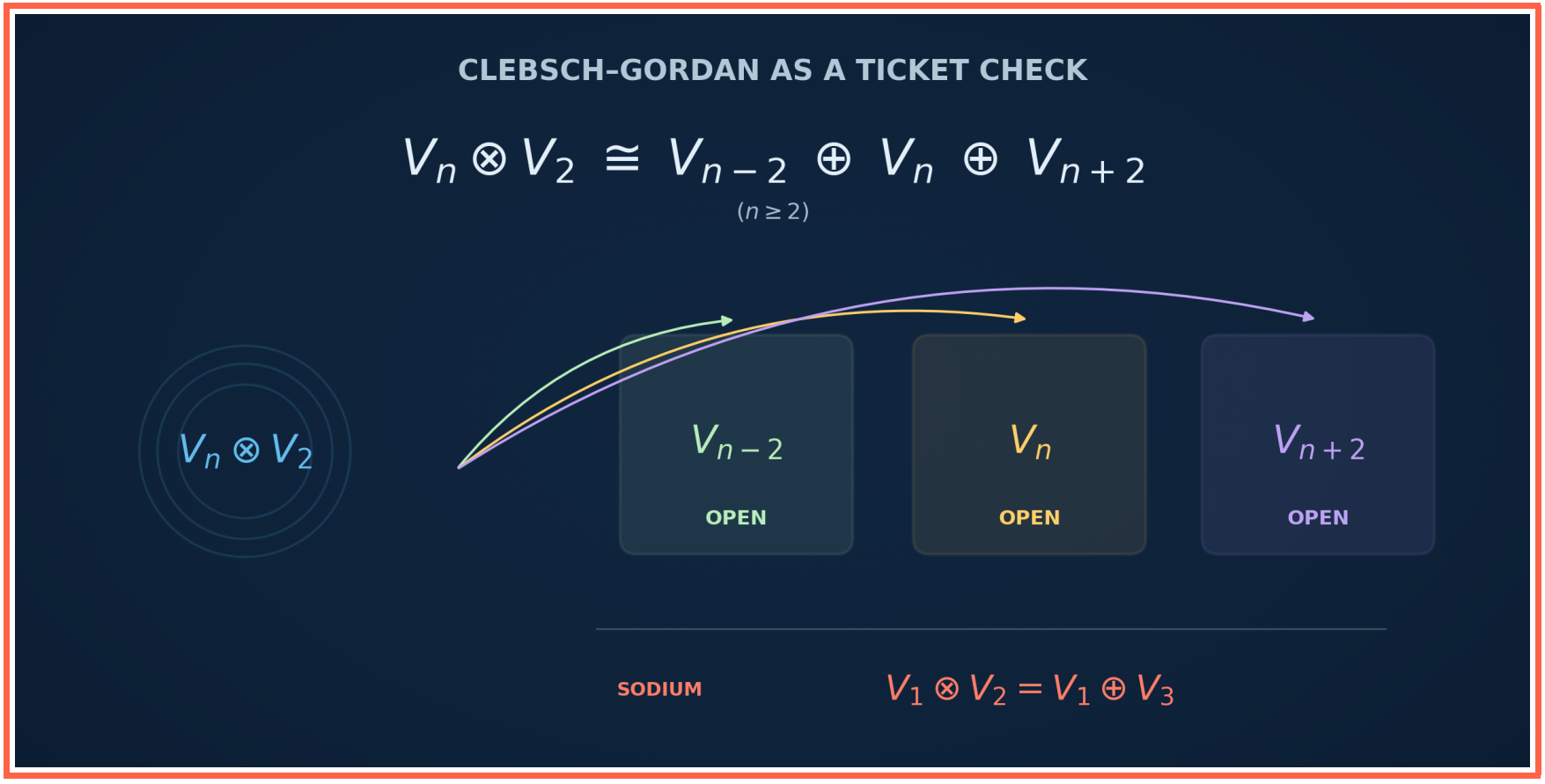
- Symmetry For an isolated atom, $SU(2)$ rotations preserve energy.
- Lie algebra $[J_x, J_y] = iJ_z$ (and cyclically; $\hbar = 1$).
- Labels As before, V_n has $n+1$ states (spin $n/2$); m labels the J_z-eigenvalue.

The dipole is a spin-1 messenger



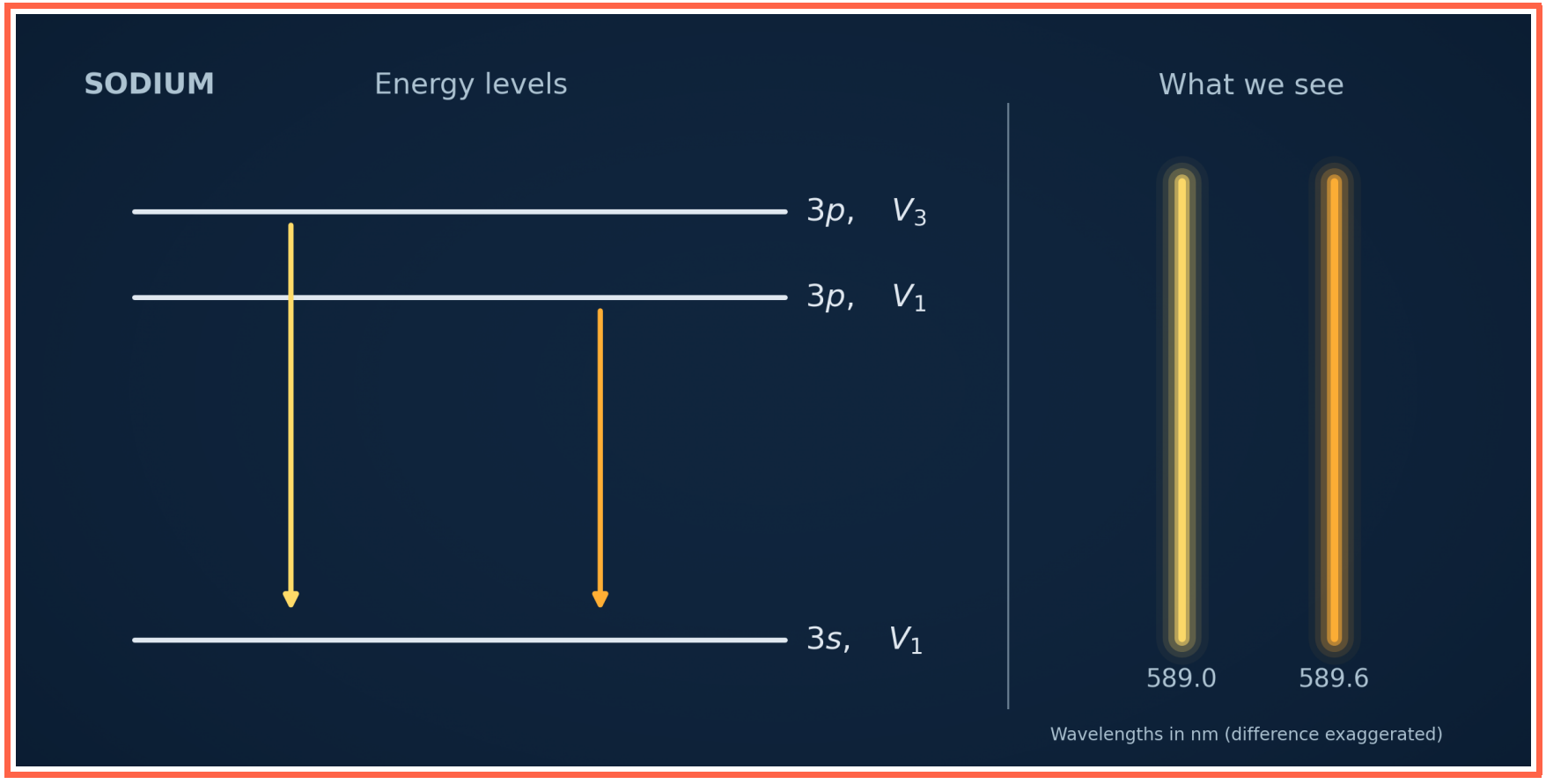
- Vector The electric dipole $\mathbf{D} = (D_x, D_y, D_z)$ transforms as V_2.
- Lie algebra Label its three components D_q ($q = -1, 0, 1$): $[J_z, D_q] = qD_q$.
- Meaning Inside a multiplet, D_q can change m by q; the label n is a different question.

A tensor product becomes a selection rule



- Tensor rule $V_n \otimes V_2 \cong V_{n-2} \oplus V_n \oplus V_{n+2}$ for $n \geq 2$.
- Sodium case $V_1 \otimes V_2 \cong V_1 \oplus V_3$.
- Meaning From V_1, the dipole can only reach V_1 or V_3.

Sodium: the representation theory pays off



- Orbital names In $3s$ and $3p$, 3 is the shell; s is round, p is two-lobed.
- $SU(2)$ $3s$ carries V_1; $3p$ splits into V_1 and V_3.
- Result Both jumps pass the tensor and parity tests (s even, p odd).

Thank you for your attention!

Next time: Robots, rotations, and 3D vision