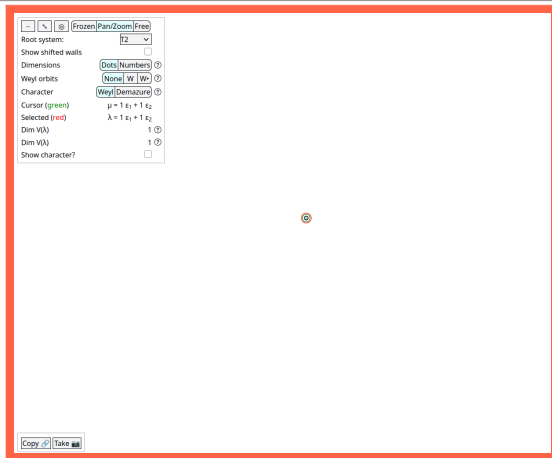


Lie theory - part 26

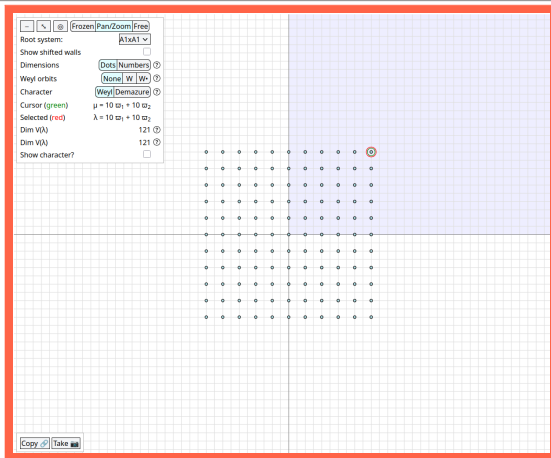
Or: Weights and weight decompositions

Restrict to the torus



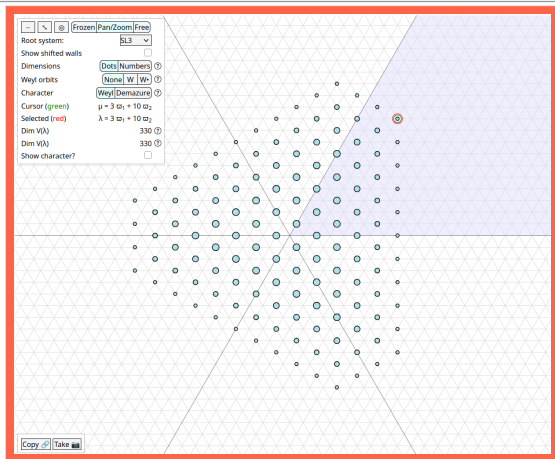
- **Start** Restrict a representation of G to a maximal torus T
- **Easy case** Elements of T commute, so diagonalise them together
- **Result** V splits into simultaneous eigenspaces

One circle, one integer



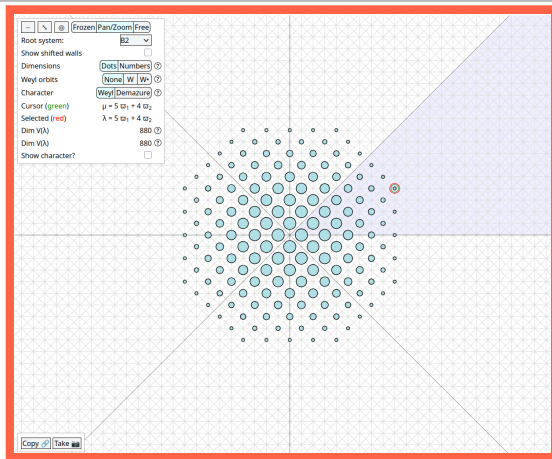
- Simplest torus Start with $T = S^1$
- On an eigenline $e^{i\theta}$ acts on an eigenline by $e^{im\theta}$, with $m \in \mathbb{Z}$
- Weight The integer m records how that circle acts

Several circles, several integers



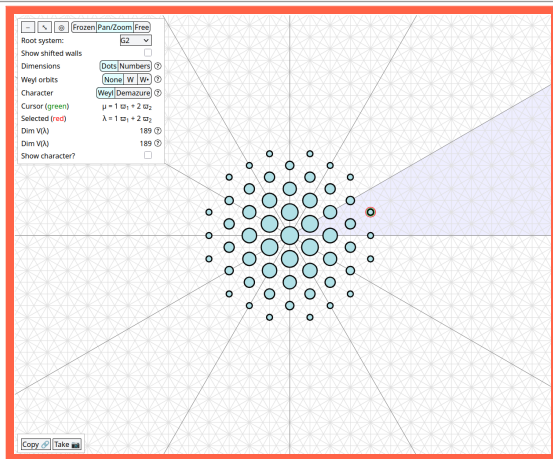
- Many circles For $T \cong (S^1)^r$, a weight records r integers
- Same weight Vectors with the same tuple form a weight space V_λ
- Decomposition The whole representation splits as $V = \bigoplus_\lambda V_\lambda$

Back to $SU(2)$



- ▶ On $\mathbb{C}^2$ $\text{diag}(e^{i\theta}, e^{-i\theta})$ has weights $+1, -1$
- ▶ Higher irreps Dimension $n + 1$: weights $n, n - 2, \dots, -n$
- ▶ Pattern Our old $SU(2)$ irreps now have visible internal structure

Why bother with weights?



- **Compression** Keep weights and multiplicities, not all the matrices
- **Tensor products** Weights add (remember Clebsch–Gordan)
- **Next** Weights for G acting on its Lie algebra are called roots

Thank you for your attention!

Next time: Root systems