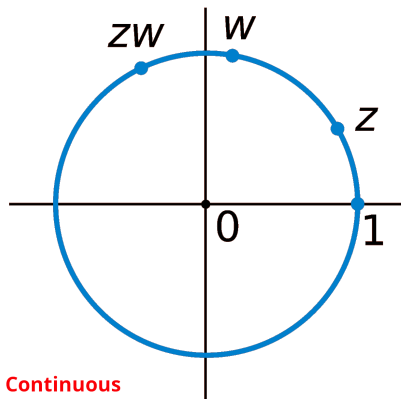
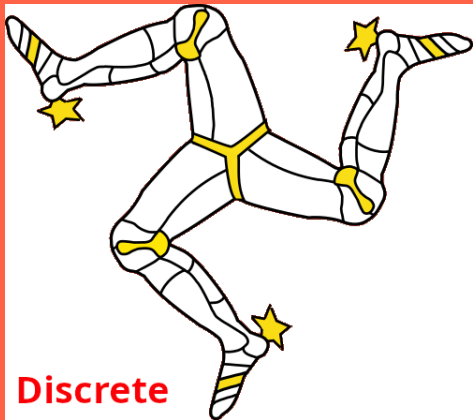


Lie theory - part 1?

Or: Introduction

What is Lie theory?



- ▶ Lie theory = the mathematics of continuous/smooth symmetry
- ▶ Two protagonists Lie groups G (global) and Lie algebras \mathfrak{g} (infinitesimal)
- ▶ Guiding slogan “Differentiate symmetry” and later “integrate it back”

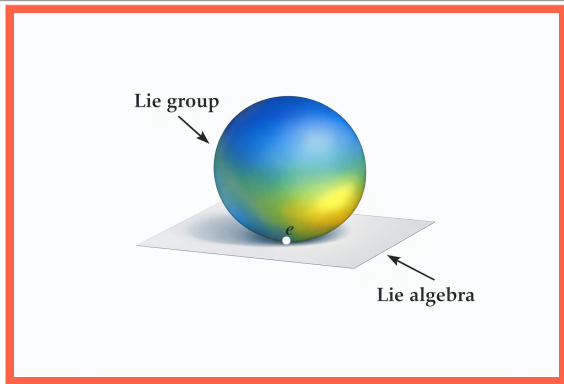
First examples: matrix Lie groups



A grid of numbers, likely representing a matrix, with a red border. The numbers are arranged in a grid pattern, with some numbers being larger and more prominent than others. The numbers are: 3.7, -1.2, 5.6, 8.0, -2.4, 1.8, -3.1, -0.8, -1.4, -2.0, 1.8, -0.6, -4.5, -0.8, -3.3, 1.2, -0.8, -0.8, -0.8, -1.3, -4.7.

-
- ▶ **Main zoo** $GL(n)$, $SL(n)$, $O(n)$, $SO(n)$, $SP(n)$, $U(n)$, $SU(n)$, and friends
 - ▶ **Why matrices?** Concrete + real intuition + no hand-waving about charts
 - ▶ **Micro-move** Define G by equations, then differentiate the equations locally

Why care?



-
- ▶ **Unreasonable effectiveness** Continuous symmetry forces structure you can actually compute
 - ▶ **Compression tool** Lie algebras turn nonlinear group questions into linear algebra (plus a bracket)
 - ▶ **Universal language** Geometry, analysis, number theory, physics, and representation theory keep meeting here

Applications (a non-exhaustive teaser)

Mathematical formulation of the Standard Model

3 languages

Article Talk

Read Edit View history Tools

From Wikipedia, the free encyclopedia

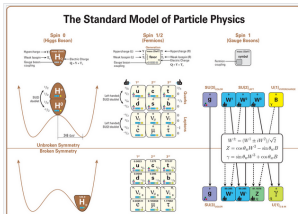
For a less mathematical description and overview, see *Standard Model*.

The **Standard Model of particle physics** is a gauge quantum field theory containing the internal symmetries of the unitary product group $SU(3) \times SU(2) \times U(1)$. The theory is commonly viewed as describing the fundamental set of particles – the leptons, quarks, gauge bosons and the Higgs boson.

The Standard Model is renormalizable and mathematically self-consistent;^[1] however, despite having huge and continued successes in providing experimental predictions, it does leave some unexplained phenomena.^[2] In particular, although the physics of special relativity is incorporated, general relativity is not, and the Standard Model will fail at energies or distances where the graviton is expected to emerge. Therefore, in a modern field theory context, it is seen as an effective field theory.

Quantum field theory [edit]

Main article: *Quantum field theory*



Standard Model of Particle Physics. The diagram shows the elementary particles of the Standard Model (the Higgs boson, the three generations of quarks and leptons, and the gauge bosons), including their names, masses, spins, charges, chiralities, and interactions with the strong, weak and electromagnetic forces. It also depicts the crucial role of the Higgs boson in electroweak symmetry breaking, and shows how the properties of the various particles differ in the (high-energy) symmetric phase (top) and the (low-energy) broken-symmetry phase (bottom).

- ▶ **Math** Representation theory, harmonic analysis, differential geometry, number theory
- ▶ **Physics** Angular momentum, quantum spin, gauge symmetries, particle physics
- ▶ **Real world** Robotics and control, 3D vision/graphics, navigation (poses), symmetry-aware machine learning

Roadmap: where the series is going



-
- ▶ **A** Foundations: matrix groups, Lie algebras, exp / log, adjoint action
 - ▶ **B** Representations: $SU(2)$ as the laboratory (characters, tensor products, etc.)
 - ▶ **C** General structure: tori, weights/roots, Weyl group, classification

Thank you for your attention!

I hope that was of some help.