

**What is...the Poincaré conjecture?**

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Or: Spheres will be spheres

1895

JOURNAL

DE

L'ÉCOLE POLYTECHNIQUE.

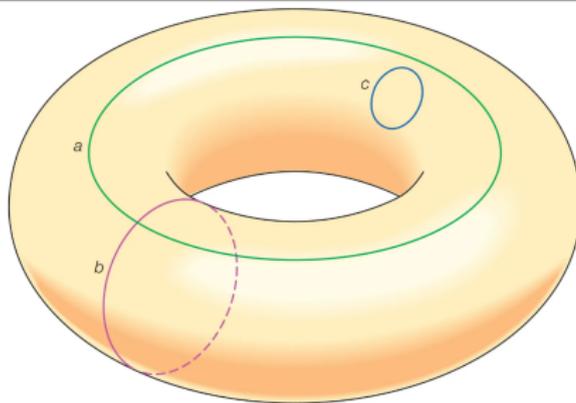
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ANALYSIS SITUS;

PAR M. H. POINCARÉ.

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- ▶ Topology started long time ago e.g. Euler, Gauss, Riemann...
  - ▶ The “real kick off” was Poincaré’s paper Analysis situs
  - ▶ In this work Poincaré created “essentially all traditional notions” of topology

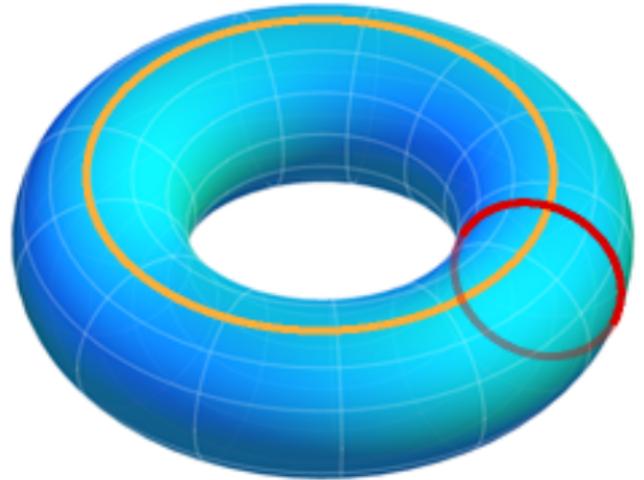
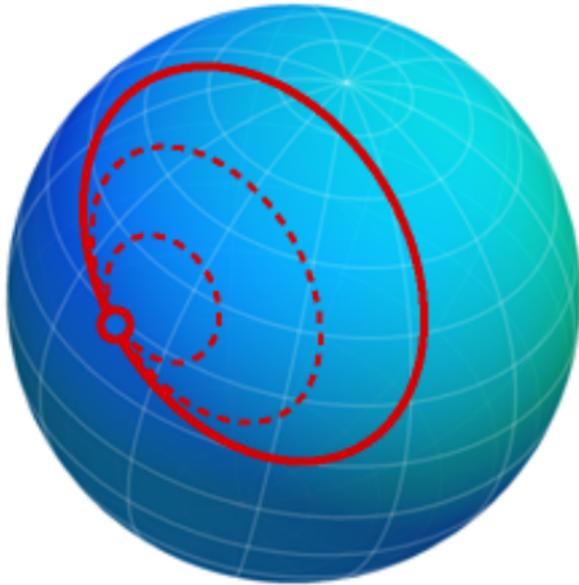
## Poincaré's big "conjecture"



- ▶ A closed 3 mfd with trivial fundamental group is homeomorphic to  $S^3$
- ▶ In the beginning, there was no conjecture as "this is obvious"
- ▶ Later it was stated with trivial homology but we have already seen Poincaré spheres; then it took the form above but formulated as a question

## Progress is always slow...

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- ▶ The existence of homology spheres shows that 3d are more complicated than 2d (Poincaré's conjecture is well-known for 2d)
  - ▶ Turns out that it is even much more complicated
  - ▶ Perelman published a proof outline in 2002–2003, a hundred years after the conjecture was made

## For completeness: A formal statement

### Generalized Poincaré conjecture

Every homotopy  $n$  sphere is isomorphic to  $S^n$  where e.g.:

- ▷ Isomorphic could mean as topological manifolds
- ▷ Isomorphic could mean as smooth manifolds

- ▶ For  $n \leq 2$  this is well-known
- ▶ This was proven by Smale for  $n > 3$  (topological)

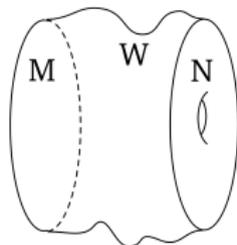
ANNALS OF MATHEMATICS  
Vol. 74, No. 2, September, 1961  
*Printed in Japan*

### GENERALIZED POINCARÉ'S CONJECTURE IN DIMENSIONS GREATER THAN FOUR

BY STEPHEN SMALE\*

(Received October 11, 1960)

(Revised March 27, 1961)



We discuss the proof in a later video

- ▶ The topological version for  $n = 3$  was proven by Hamilton–Perelman

## The smooth case is different

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Dimension	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Smooth types	1	1	1	$\geq 1$	1	1	28	2	8	6	992	1	3	2	16256	2	16	16	523264	24

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- ▶ The *e.g.*  $n = 7$  case of the smooth Poincaré conjecture is wrong
  - ▶  $n = 4$  case of the smooth Poincaré conjecture is widely open

**Thank you for your attention!**

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I hope that was of some help.