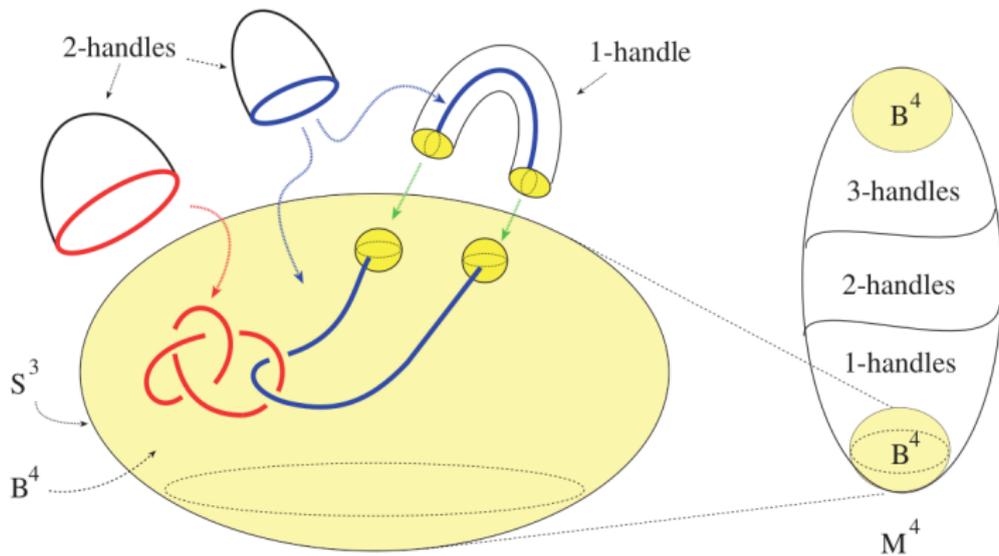


What is...4d Kirby calculus again?

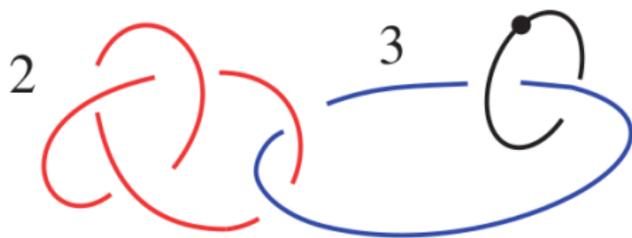
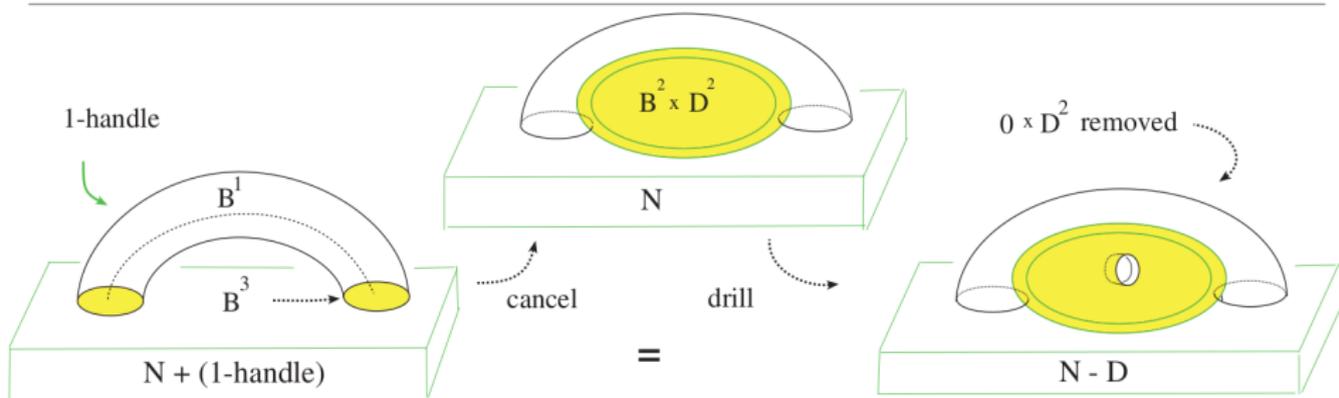
Or: Knots and four manifolds, part 2

Reminder: 4d Kirby diagrams



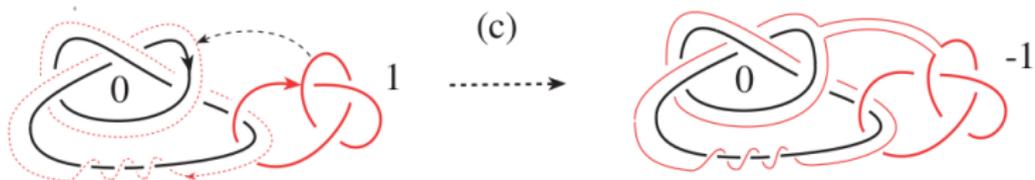
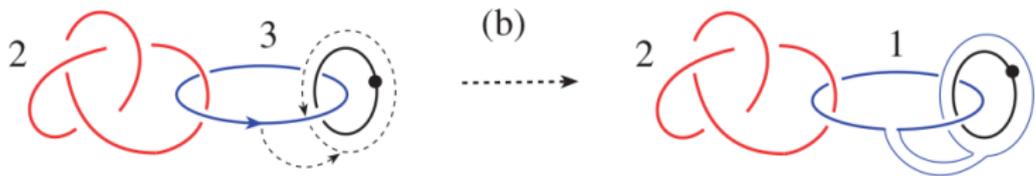
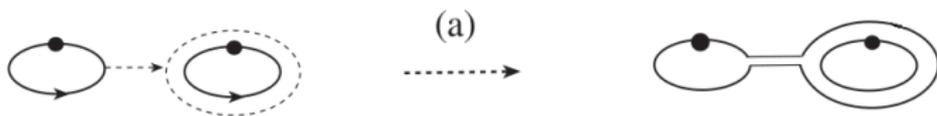
- ▶ The attaching sphere for a 1 handle is δD^1 Two points/two balls
- ▶ The attaching sphere for a 2 handle is δD^2 A (framed) knot
- ▶ The above picture is a Kirby presentation of a 4mfd M

Simplifying diagram



- ▶ Attaching a 1 handle is the same as “drilling” a disk D^2 out of the 0 handle
- ▶ Drilling = push int disk into int M and remove a tubular neighborhood
- ▶ Instead of drawing 2 balls we can draw one **drill disc** (we put a dot on them)

Handle slide relations

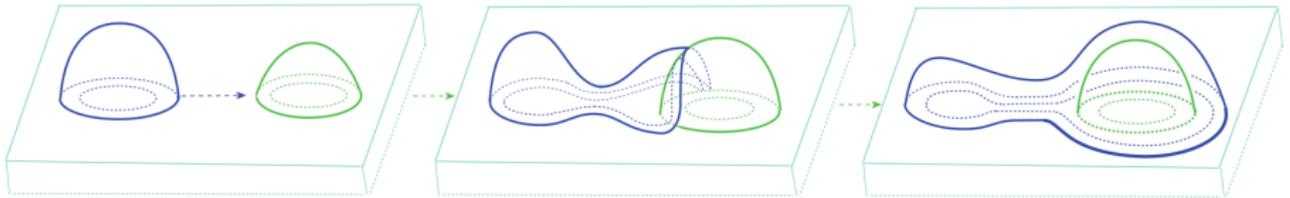


- ▶ Without changing M we can slide k handles over r handles for $k \geq r$
- ▶ The result for 1-over-1, 2-over-1 and 2-over-2 are above
- ▶ These are relations among 4d Kirby diagrams

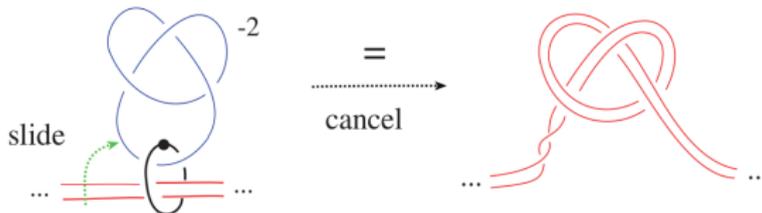
For completeness: A formal statement

Cerf theory Any two relative handle decompositions of M are related by the moves below (not quite a 4d Kirby calculus)

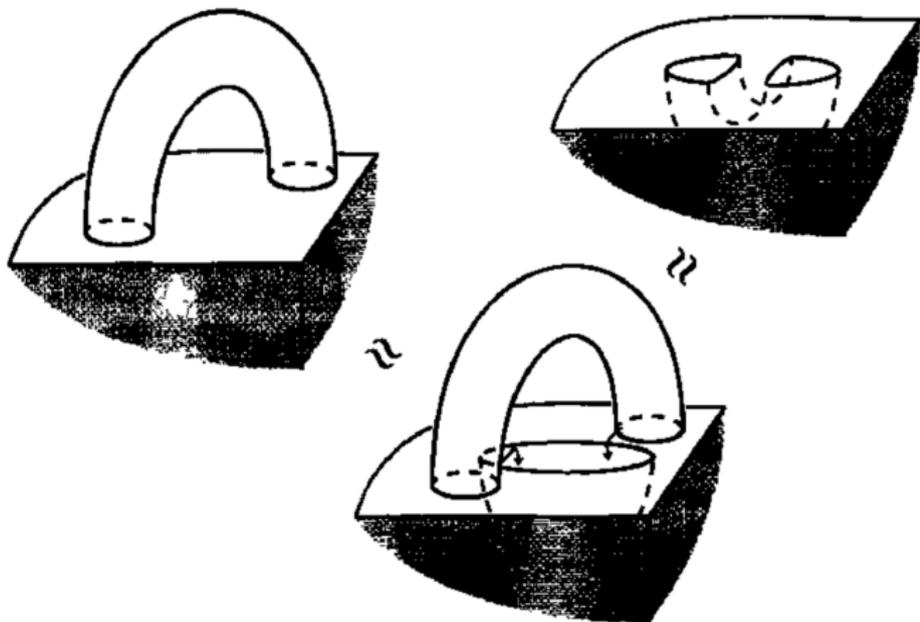
- ▶ **Isotopies** are e.g. Reidemeister moves II + III
- ▶ **Handle slides** as on the previous page



- ▶ **Handle pair creation/cancellation** is e.g.:



Drills = tunnels in 3d (for comparison)



-
- ▶ In 3d we have that 1 handles are bridges
 - ▶ Bridges are tunnels
 - ▶ Tunnels are obtained by “drilling” a disc D^2 into M^3

Thank you for your attention!

I hope that was of some help.