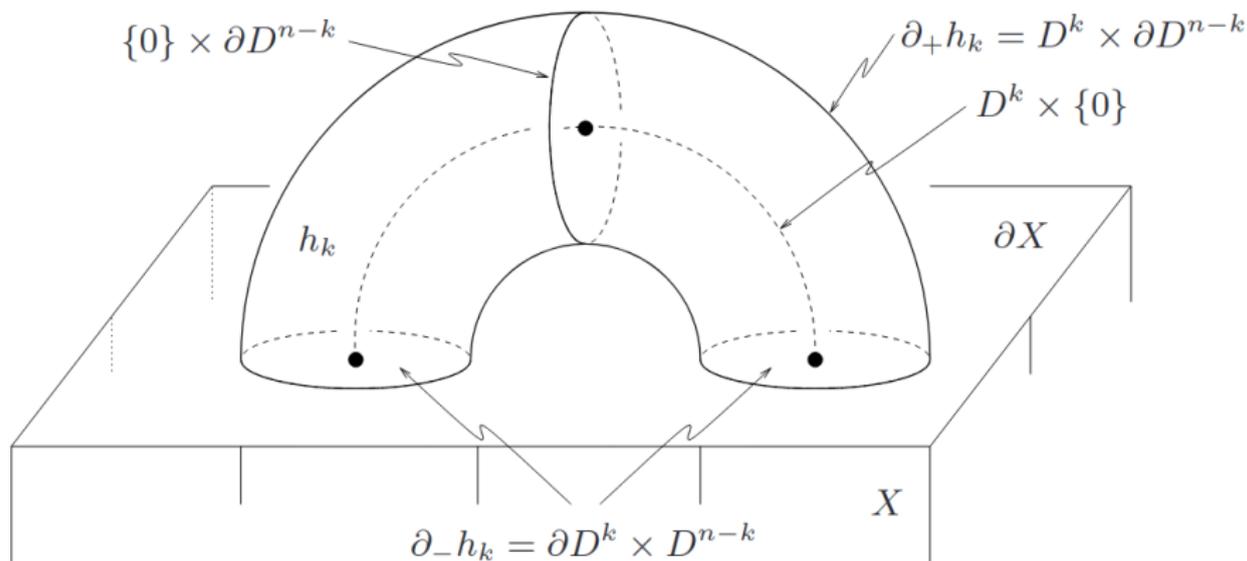


What is...4d Kirby calculus?

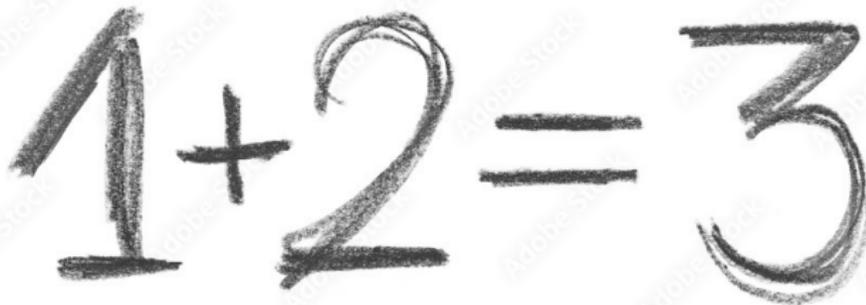
Or: Knots and four manifolds, part 1

Attaching handles



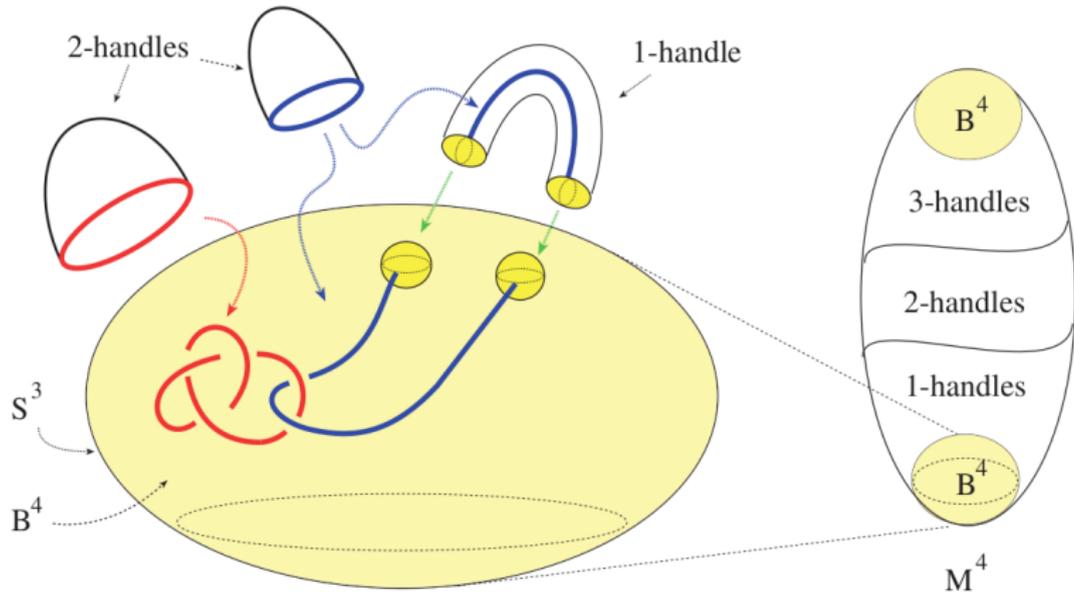
- ▶ **Handle decomposition** \iff attaching k handles to an n mfd X
- ▶ **k handle** $= h^k = D^k \times D^{n-k}$
- ▶ Attaching **depends on k** , dimension is always n

$$1+2=3$$

A hand-drawn equation $1+2=3$ written in black chalk on a white background. The numbers and symbols are slightly blurred and have a textured appearance. The '1' is a simple vertical stroke, the '+' is a small cross, the '2' has a curved top, the '=' consists of two parallel horizontal lines, and the '3' has a curved bottom.

-
- ▶ To construct 4mfds we a priori need 0,1,2,3,4 handles
 - ▶ 0 and 4 handles are just “opening-closing” so we can ignore them
 - ▶ Theorem 3 handles are determined by 1 and 2 handles
 - ▶ Hence, we effectively need only 1 and 2 handles

Knots again!

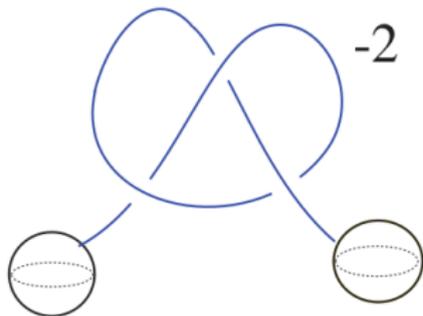


- ▶ The attaching sphere for a 1 handle is δD^1 Two points/two balls
- ▶ The attaching sphere for a 2 handle is δD^2 A (framed) knot
- ▶ The above picture is a Kirby presentation of a 4mf

For completeness: A formal statement

Any closed orientable smooth/piecewise-linear 4-mfd admits a Kirby presentation

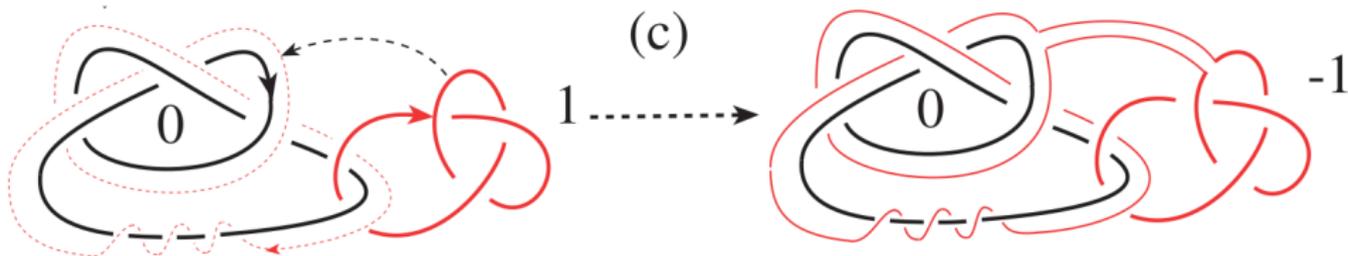
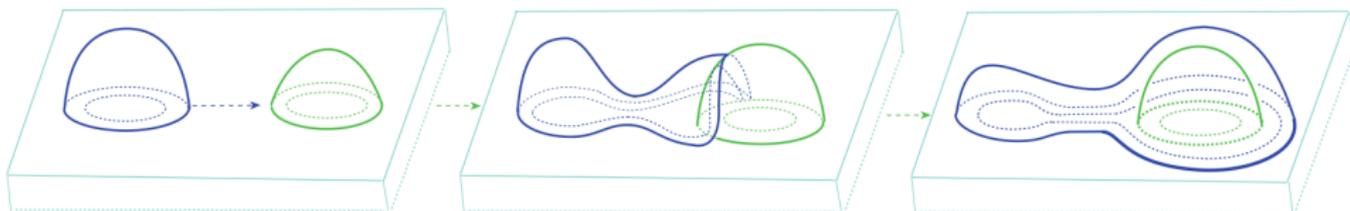
- ▶ Note that one can stack 2 handles on 1 handles
- ▶ The diagram you see in a Kirby presentation is a 4d Kirby diagram



4d Kirby diagrams consists of pairs of balls for 1 handles and of framed=numbered knots for 2 handles

- ▶ There are some relations among 4d Kirby diagrams, e.g. handle slides

The handle slides again



- ▶ One can slide a k handle over another r handle when $r \leq k$
- ▶ **Top** Sliding a 2 handle over a 2 handle
- ▶ **Top** Sliding a 2 handle over a 2 handle in 4d Kirby diagrams

Thank you for your attention!

I hope that was of some help.