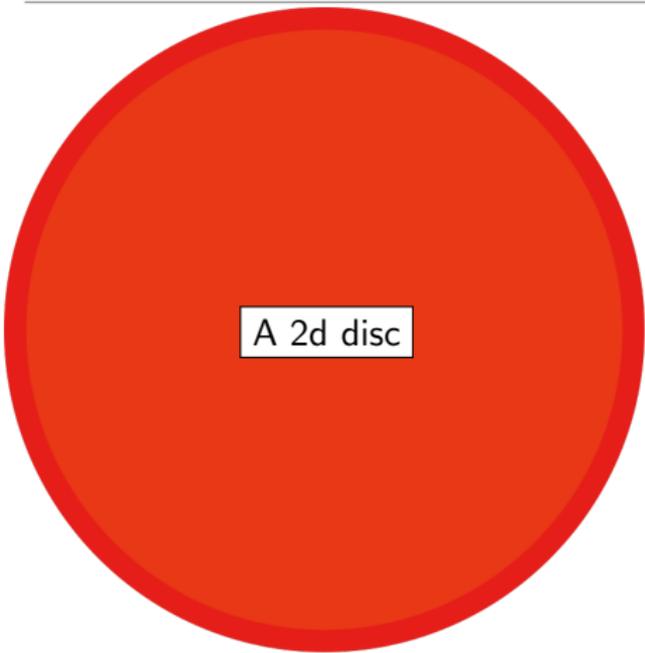


**What are...three manifolds?**

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Or: A glimpse of 4d

## 2d discs versus 3d discs



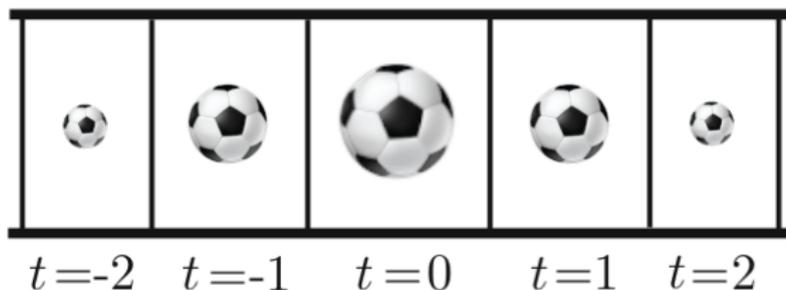
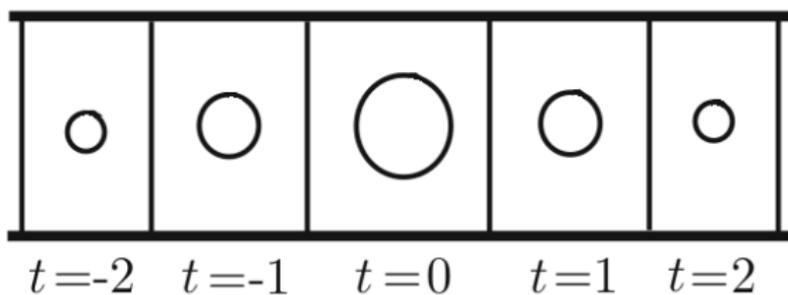
A 2d disc



A 3d disc

- ▶ Recall that a surface was locally made of discs
- ▶ 3mfds are locally made of solid balls
- ▶ Example Three space  $\mathbb{R}^3$  is a 3mfd, but its not closed

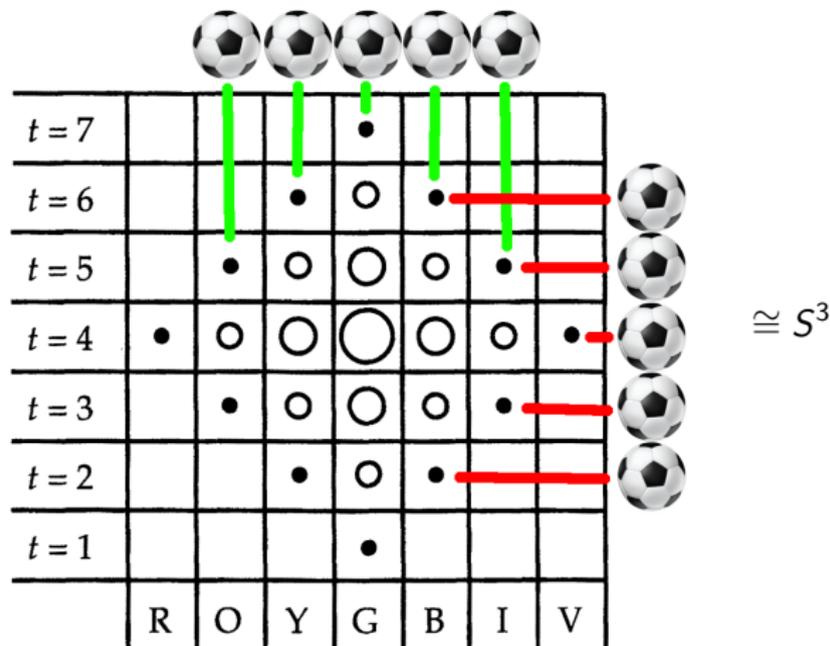
## Sorry, it will be 4d



= 3d sphere  $S^3$

- ▶ A closed surface (sphere, torus, ...) is 2d but needs at least  $\mathbb{R}^3$  to be realized
- ▶ A 3mfd is a 3d version of a surface and we expect 4d to show up

## More movies



- ▶ We can also think of a 3mfd as a **double movie**
- ▶ These pictures are great for **certain** 3mfds
- ▶ However, in **general** one needs a better way of thinking about them!

## For completeness: A formal definition

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A closed 3mfd  $M$  is a topological spaces such that:

- (i) Every  $x \in M$  has an open neighborhood  $\cong$  to  $(X \subset \text{Euclidean 3-space})$  open 3d discs and  $M$  is compact
  - (ii)  $M$  is nonempty, second-countable, and Hausdorff Technical assumptions
- 

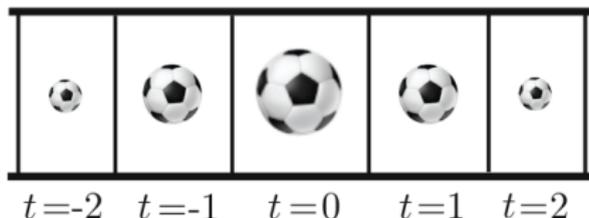
A 3mfd  $M$  with boundary is a topological spaces such that:

- ▶ Every  $x \in M$  has an open neighborhood  $\cong$  to  $(X \subset \text{closure of upper half-space})$  open 3d discs or 3d half-discs
  - ▶  $M$  is nonempty, second-countable, and Hausdorff Technical assumptions
- 

### Examples

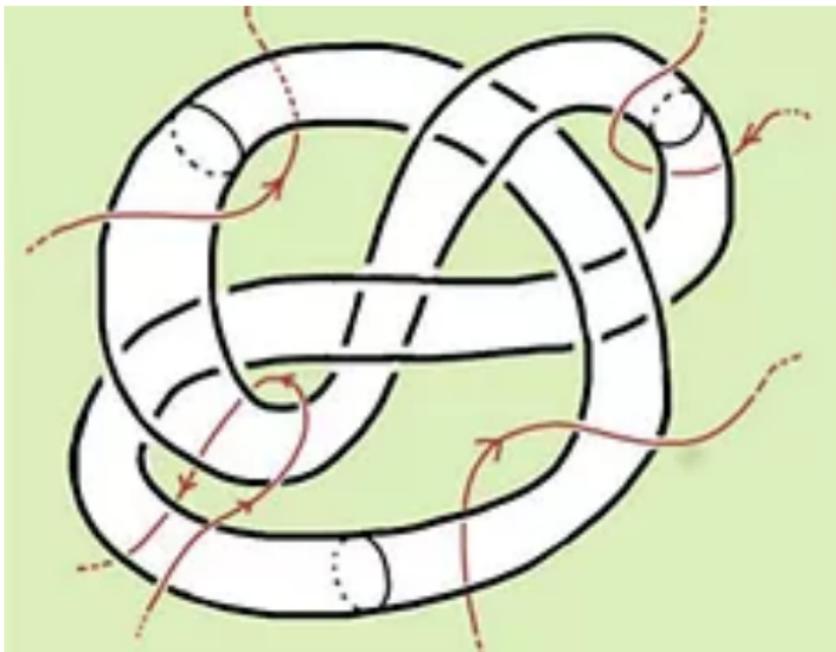
Left closed

Right with boundary



## 3mfds and knots

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- ▶ We have already seen **knot complements** and these are 3mfds with boundary
  - ▶ In general, we will see that 3mfds and knots are **closely related**

**Thank you for your attention!**

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I hope that was of some help.