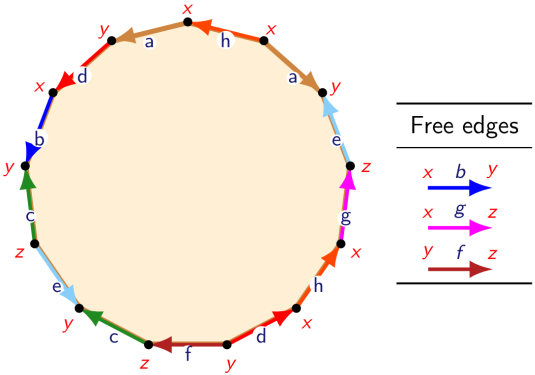
What is...the classification of surfaces?

Or: Punctures, handles, projective planes

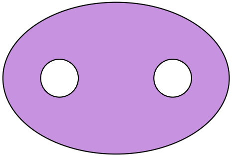
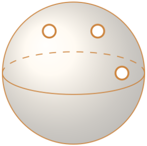
Polygons and words

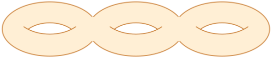
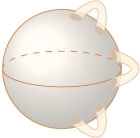


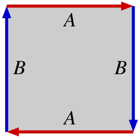
$adb\overline{c}e\overline{c}\overline{f}dhge\overline{a}h$

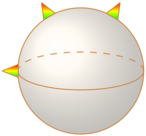
- Every surface word gives a surface; every surface gives many surface words
- There is some ambiguity with surface words *e.g.* via many free edges describing one boundary
- This time How to go from a word to a normal form of a surface

Punctures, handles, projective planes

$S^2\#(\#^3 D^2)$:  $\cong$ 

$S^2\#(\#^3 T)$:  $\cong$ 

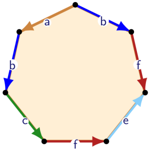
$S^2\#(\#^3 \mathbb{R}P^2)$: $\Bigg($  $+ 3x$ 

$\Bigg)$ $\cong$ 

- Standard surfaces: punctures, handles, projective planes
- Every surface is obtained by combining these

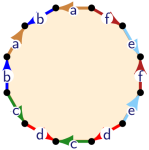
Standard words for standard surfaces

$S^2\#(\#^3 D^2)$:



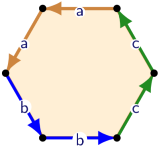
$a\ b\ c\ f\ e\ \overline{f}\ \overline{b}$

$S^2\#(\#^3 T)$:



$= a\ b\ \overline{a}\ \overline{b}\ c\ d\ \overline{c}\ \overline{d}\ e\ f\ \overline{e}\ \overline{f}$

$S^2\#(\#^3 \mathbb{R}P^2)$:



$= a\ a\ b\ b\ c\ c$

- The above are the standard words for these
- Every surface can be written combining these

For completeness: A formal statement

Every surfaces S is of the form

$$S \cong (\#^d D)\#(\#^h T)\#(\#^p \mathbb{R}P^2), d \in \mathbb{N}, h \in \mathbb{N}, p \in \{0,1\}$$

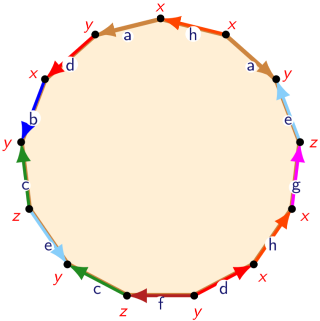
S is completely determined by:

(i) Its number of boundary components Punctures

(ii) Its Euler characteristic $\chi = |V| - |E| + |F|$ "Handles"

(iii) Whether it is orientable or not Projective planes

Example



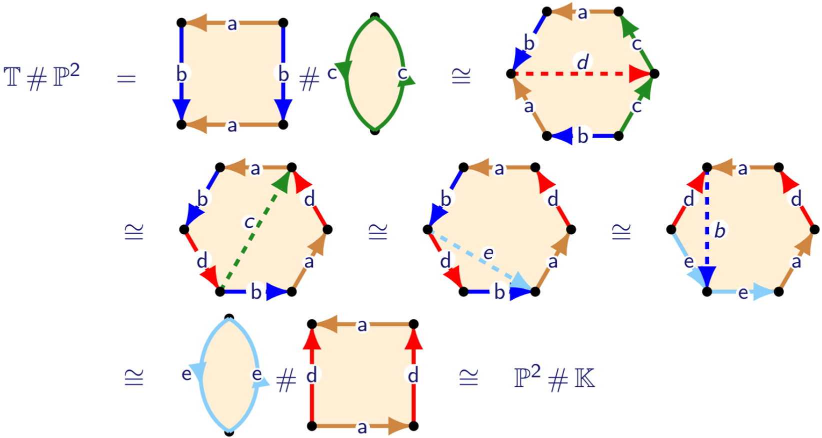
$|V| = 3, |E| = 8, |F| = 1, \chi = -4$

One boundary component

A pair $dd \Rightarrow$ non-orientable

For S we have $(d, h, p) = (1, 2, 1)$

Projective planes are weird...



- Gluing a handle to $\mathbb{R}P^2$ is the same as gluing a Klein bottle to it
- Klein bottle ≅ two projective planes glued together
- We can thus decide whether we like more handles or projective planes

Thank you for your attention!

I hope that was of some help.