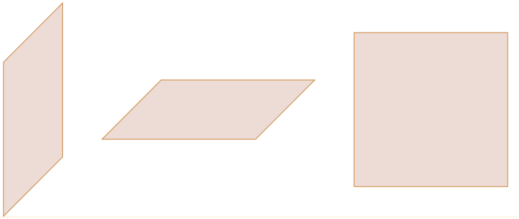
What are...surfaces?

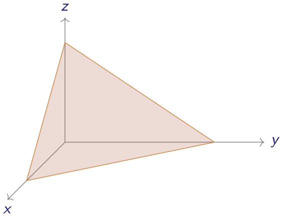
Or: Spheres and friends

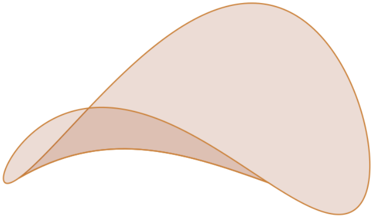
Locally a disc - again

- Surface without boundary "=" every point that has a neighborhood homeomorphic to a disc
- Surface with boundary "=" as before but with potential boundary points
- Boundary point = local neighborhoods homeomorphic to a half-disc
- The torus = swim ring is a closed (without boundary) surface
- A pair of pants is a surface with boundary

Many rectangles

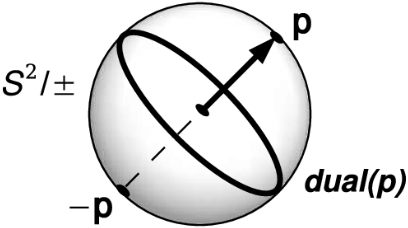
parts of coordinate planes: 

simplex: 

, curvy: 

- Listing all surfaces is hopeless, *e.g.* there are already ∞ many "rectangles"
- But all of these are the same surface up to $\cong$
- Goal (ambitious?) Classify surfaces up to $\cong$

More surfaces



- There are way more surfaces: sphere, torus, $\mathbb{R}P^2$, Klein bottle, ...
- It is not even clear whether they are homeomorphic or not
- We need some way of listing surfaces efficiently

For completeness: A formal definition

A closed surfaces S is a topological spaces such that:

(i) Every $x \in S$ has an open neighborhood $\cong$ to ($X \subset$Euclidean plane) open Discs and S is compact

(ii) S is nonempty, second-countable, and Hausdorff Technical assumptions

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A surfaces S with boundary is a topological spaces such that:

(i) Every $x \in S$ has an open neighborhood $\cong$ to ($X \subset$closure of upper half-plane) open Discs or half-discs

(ii) S is nonempty, second-countable, and Hausdorff Technical assumptions

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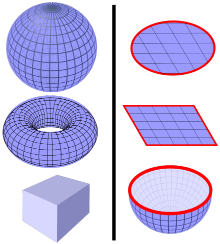
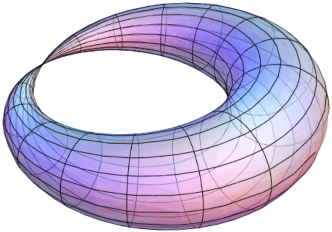
Examples

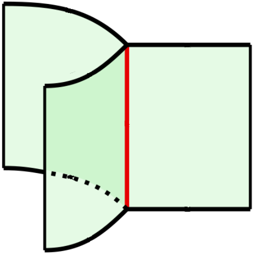
Left closed

Right with boundary



Non-examples





- The top has a cusp ⇒ not a surface
- The bottom has three-fold singularity ⇒ not a surface

Thank you for your attention!

I hope that was of some help.