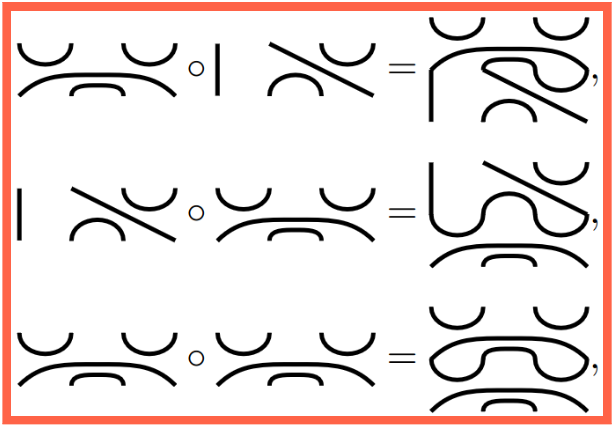
Diagram algebras - part 9

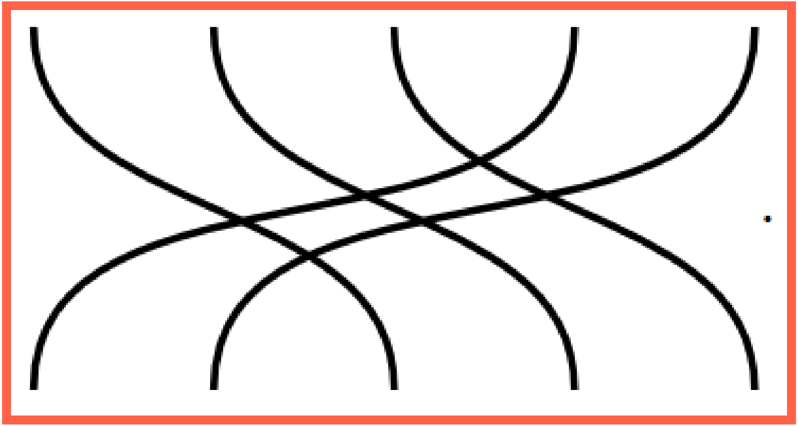
Or: Brauer joins the party

Previously: pairings were enough



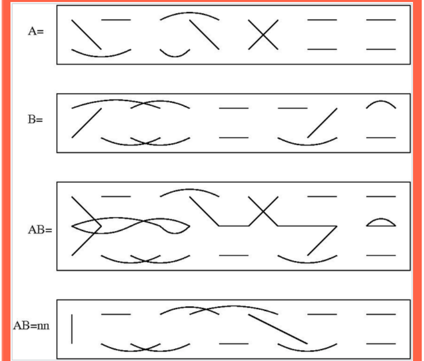
- Previously Rumer–Teller–Weyl showed that pairings already see SL_2
- Example Cups and caps are not decoration; they are invariant tensors
- Vibe Temperley–Lieb is the planar corner of a much larger story

Then crossings walked in



- Crossing A crossing is the basic swap of two tensor slots
- Storytime Schur–Weyl duality says these swaps organize the GL_t side
- Example A little braid picture can simply mean: permute the factors

Brauer's move: use both



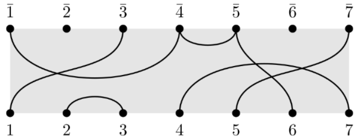
The product of 2 basis elements A and B of the Brauer algebra with $n = 12$

- **Recipe** Keep the pairings from TL and add the crossings from permutations
- **Brauer** This mixed calculus sees orthogonal and symplectic symmetry
- **Example** Strands may cross, pair up, and close into loops while stacking

Jones and Martin: bigger blocks

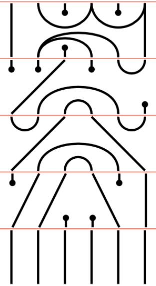
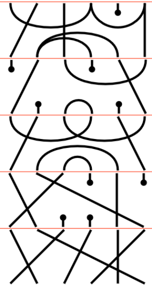
Diagrams [edit]

A partition of $2k$ elements labelled $1, \bar{1}, 2, \bar{2}, \ldots, k, \bar{k}$ is represented as a diagram, with lines connecting elements in the same subset. In the following example, the subset $\{\bar{1}, \bar{4}, \bar{5}, 6\}$ gives rise to the lines $\bar{1} - \bar{4}, \bar{4} - \bar{5}, \bar{5} - 6$, and could equivalently be represented by the lines $\bar{1} - 6, \bar{4} - 6, \bar{5} - 6, \bar{1} - \bar{5}$ (for instance).



- **Upgrade** Points no longer need to be paired; they can form larger blocks
- **Storytime** Jones and Martin showed that bigger blocks can even reach S_t
- **Example** Two top dots and one bottom dot can belong to the same little clan

The history lesson in one cartoon

Symbol	Diagrams	G	V	$\mathbb{Z}$?	Symbol	Diagrams	G	V	$\mathbb{Z}$?
$pPa_{\mathbf{c}}(t)$		$SL2$	$\mathbb{C}^2 \otimes \mathbb{C}^2$	Y	$Pa_{\mathbf{c}}(t)$		St	$\mathbb{C}^t$	?
$Mo_{\mathbf{c}}(t)$		$SL2$	$\mathbb{C}^2 \oplus \mathbb{C}$	Y	$RoBr_{\mathbf{c}}(t)$		Ot, SPt	$\mathbb{C}^t \oplus \mathbb{C}$	Y*
$TL_{\mathbf{c}}(t)$		$SL2$	$\mathbb{C}^2$	Y	$Br_{\mathbf{c}}(t)$		Ot, SPt	$\mathbb{C}^t$	Y*
$pRo_{\mathbf{c}}(t)$		$GL2$	$\mathbb{C} \oplus \mathbb{C}^{det}$	Y	$Ro_{\mathbf{c}}(t)$		GLt	$\mathbb{C}^t \oplus \mathbb{C}$	Y
1		$SL2$	$\mathbb{C}$	Y	$S_{\mathbf{c}}(t)$		GLt	$\mathbb{C}^t$	Y

- TL Planar pairings are the SL_2 baby case
- Brauer Crossings plus pairings point toward O_t and Sp_t
- Next Next time we connect this cartoon to Schur–Weyl more carefully

Thank you for your attention!

I hope that was of some help.