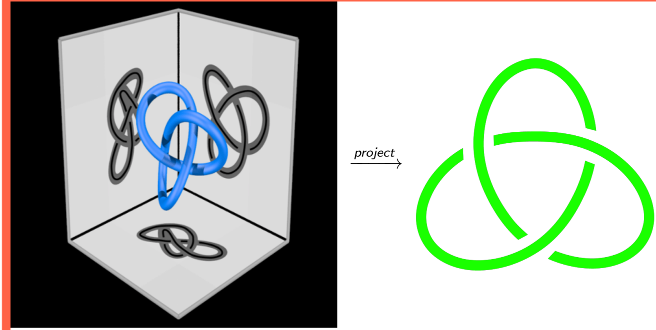
Diagram algebras - part 7

Or: How TL reads knots

Knots are local pictures



Question Do two projections represent the same knot?

- Diagram A knot drawing is built from small crossing boxes
- TL lens Each box acts on boundary points, just like a TL diagram
- Goal Turn the whole knot into one diagram-algebra calculation

Crossings are not TL, but...

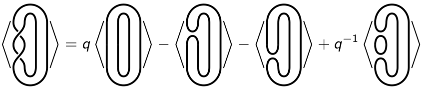
On the back of an envelope – the Kauffman bracket

(a) $\langle \emptyset \rangle = 1$ Normalization

(b) $\langle \bigcirc \cup L \rangle = (q + q^{-1}) \cdot \langle L \rangle$ Pulling out circles

(c) Kauffman Skein

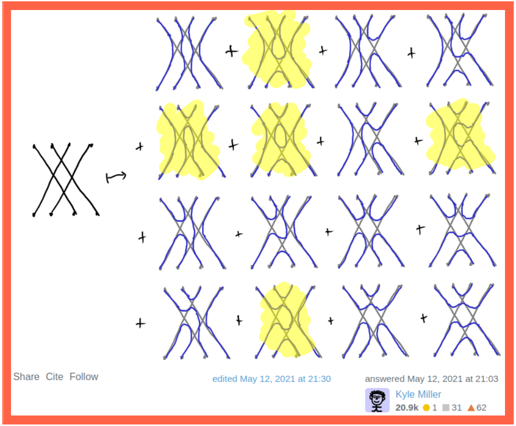
This gives the Jones polynomial up to normalization



$$= q(q + q^{-1})^2 - 2(q + q^{-1}) + q^{-1}(q + q^{-1})^2 = q^3 + q + q^{-1} + q^{-3}$$

- **Crossing** TL wants non-crossing connections, so crossings must be rewritten
- **Smoothings** One crossing becomes a tiny weighted sum of two TL pictures
- **Example** A Hopf link gives a short sum of stacked TL diagrams

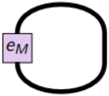
Now TL takes over



- Basis The states are non-crossing pairings of boundary points
- Product Stack the pieces exactly like in the TL algebra
- Loops Closed circles disappear and leave one fixed scalar

Close the TL calculation

Jones' construction ∼1983

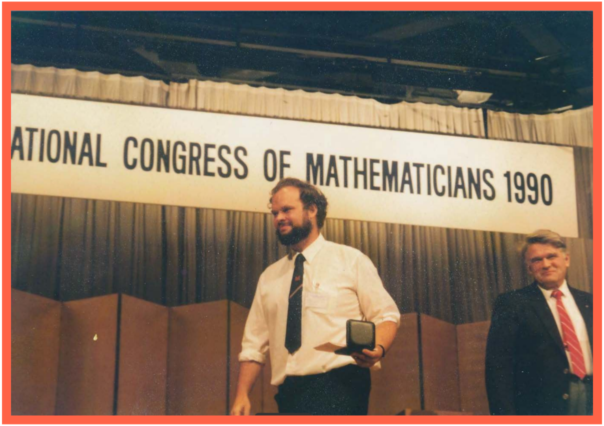
$[M:N] \rightsquigarrow$  $\in \mathbb{R}_{\geq 0}$

- Subfactor $N \subset M$, M is a N-module by left multiplication
- Assume that M is finitely generated projective N-module
  The index $[M:N] \in \mathbb{R}_{\geq 0}$ is the trace of the idempotent e_M for M
- Jones' index theorem ∼1983 The index is an invariant of $N \subset M$ and

$$[M:N] \in \{4\cos^2(\tfrac{\pi}{k+2}) | k \in \mathbb{N}\} \cup [4, \infty]$$

- Cut Open the knot into a tangle, hence into a TL element
- Trace Close the boundary points again to get a knot number
- Bracket The Kauffman bracket is this TL trace viewpoint

Why this is the Jones story



- Jones Braid crossings can be replaced by TL elements
- Trace Closing the braid and taking a trace gives a knot invariant
- Kauffman The bracket later made this look like local smoothings

Thank you for your attention!

I hope that was of some help.