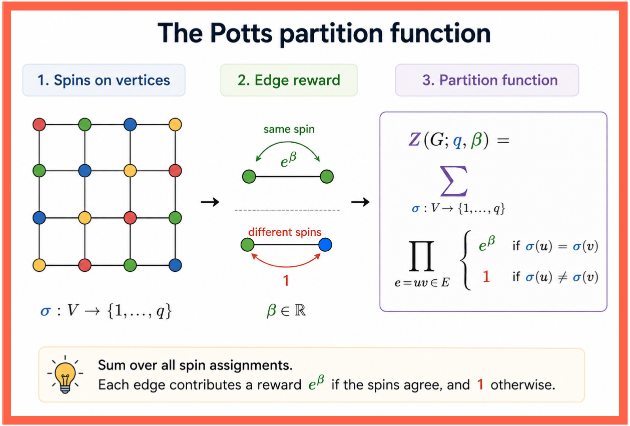
Diagram algebras - part 6

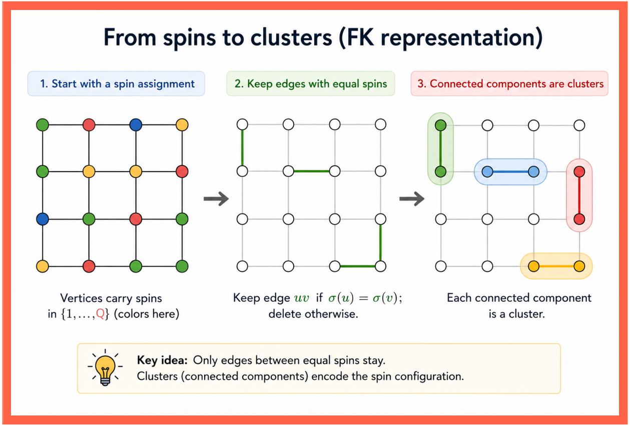
Or: Potts to TL

The Potts partition function



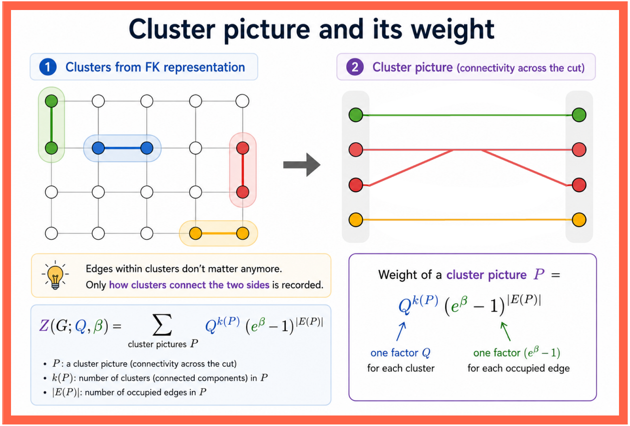
- **Spins** Put $\sigma_v \in \{1, \dots, Q\}$ at each vertex v
- **Edges** Reward agreement by $1 + t\,\delta_{\sigma_u,\sigma_v}$
- **Sum** $Z = \sum_\sigma \prod_{uv \in E}(1 + t\,\delta_{\sigma_u,\sigma_v})$ (the famous partition function)

First rewrite: clusters



- **Expand** Choose the $t\,\delta$ term on a subset $A \subseteq E$
- **Clusters** Equalities along A make connected components monochromatic
- **Formula** $Z_G(Q,t) = \sum_{A\subseteq E} t^{|A|} Q^{k(A)}$

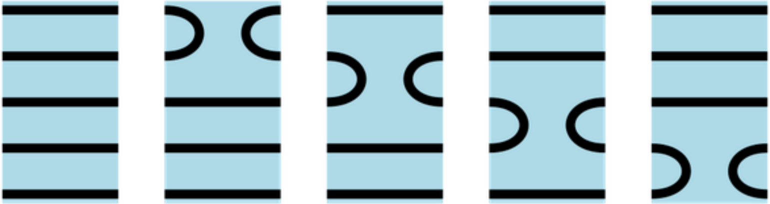
Second rewrite: loops



- **Planar** Draw boundaries between clusters and their complement
- **Weight** After a standard normalization, each loop gets d with $d^2 = Q$
- **Loop sum** A loop picture L contributes $x^{|L|} d^{\ell(L)}$

Where TL is hiding

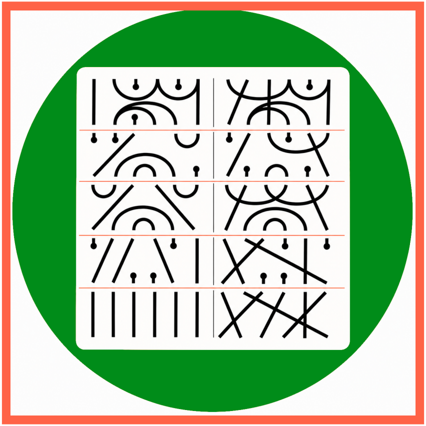
The generators of $TL_5(\delta)$ are:



From left to right, the unit 1 and the generators

- ▶ Slice Cut the lattice into strips; each strip updates boundary connectivities
- ▶ Generator A local connection is the TL generator e_i
- ▶ Relation Closing a little loop gives $e_i^2 = d\, e_i$

What the example teaches



- Potts Colored spins are rewritten as subsets $A \subseteq E \Rightarrow$ diagrams
- TL is "exactly" the algebra of stacking these local loop updates
- Well Actually the "correct" algebra is the partition algebra, but anyway

Thank you for your attention!

I hope that was of some help.