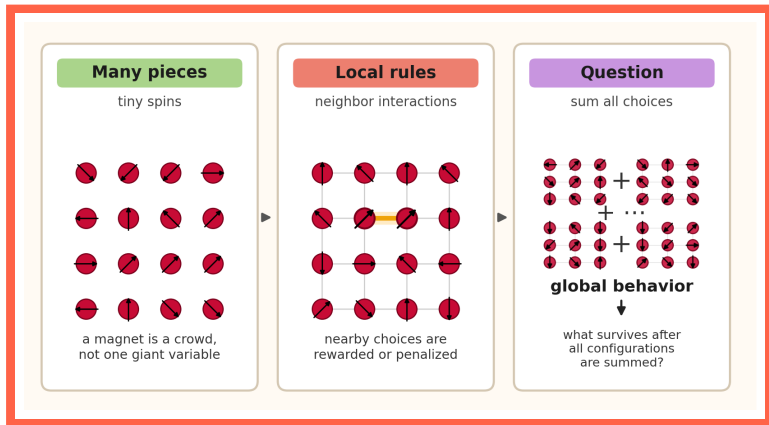


Diagram algebras - part 5

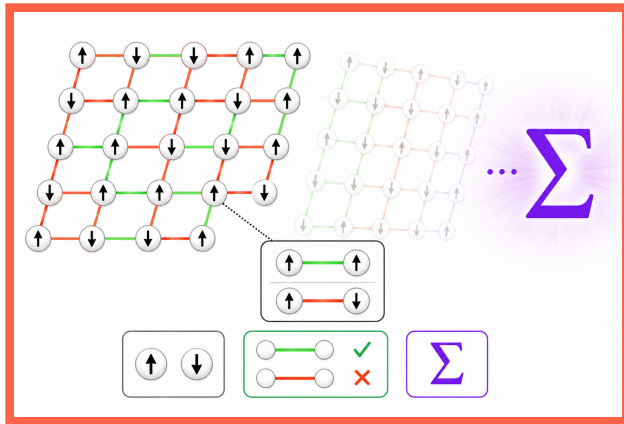
Or: Statistical mechanics wants bookkeeping

What is statistical mechanics?



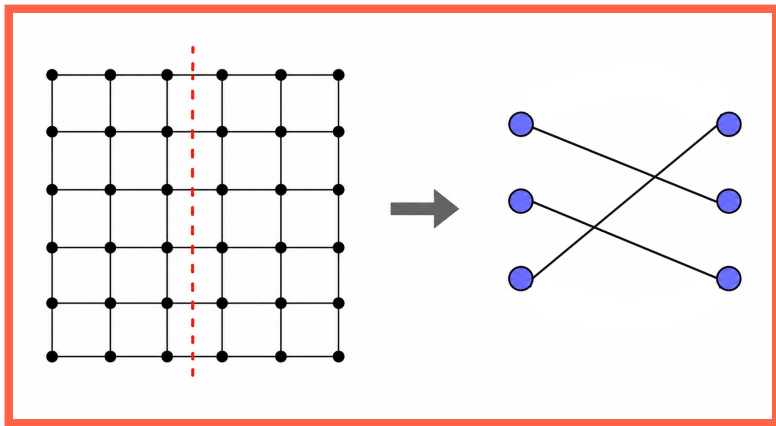
- ▶ **Many pieces** A magnet has many tiny spins, not one giant variable
- ▶ **Local rules** Neighboring pieces interact; far-away effects come from repetition
- ▶ **Question** What does the whole system do after all local choices are summed?

A toy picture: spins on a grid



- **Spins** Each site chooses up/down (Ising), or one of several colors (Potts)
- **Edges** Each edge checks whether two neighboring sites like each other
- **Sum** The answer is a huge weighted count of possible worlds

Why diagrams enter



- ▶ **Boundary** When we cut the grid, only boundary connections remain visible
- ▶ **Glue** Adding the next row glues new local pieces to old boundary data
- ▶ **Algebra** Gluing rows is multiplication, so pictures become operators

Storytime: TL did not fall from the sky

Proc. Roy. Soc. Lond. A. **322**, 251–280 (1971)

Printed in Great Britain

Relations between the ‘percolation’ and ‘colouring’
problem and other graph-theoretical problems associated with
regular planar lattices: some exact results for
the ‘percolation’ problem

BY H. N. V. TEMPERLEY

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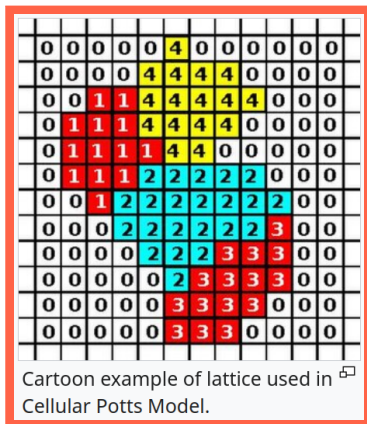
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(Communicated by S. F. Edwards, F.R.S.—Received 11 November 1970)

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- ▶ **History** TL appeared while studying lattice models in physics (Potts model)
 - ▶ **Surprise** The same rules later became a clean algebra of planar diagrams
 - ▶ **Moral** The diagrams were useful because the models already wanted them

Potts primer



- **Spins** Each site chooses one of q colors; $q = 2$ gives the Ising model
- **Physics** Equal neighboring colors are rewarded: magnets, phase transitions, ...
- **Bridge** Expanding the sum turns colors into clusters, then planar clusters into loops

Thank you for your attention!

I hope that was of some help.