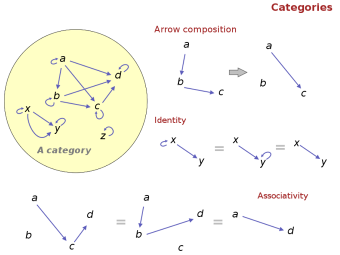
Diagram algebras - part 3

Or: Objects, morphisms, and stacking

From one algebra to a category

Category theory could also be called Arrow theory



- In FC and C Arrows $\leftrightsquigarrow$ Relations between things
- SET Vertices=sets, arrows=maps=relations between sets
- Misnomer Categories are usually named after the vertices not their arrows

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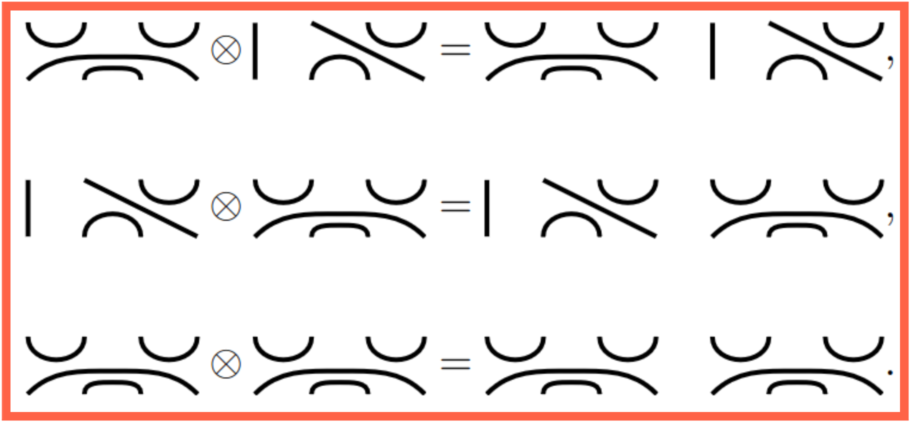
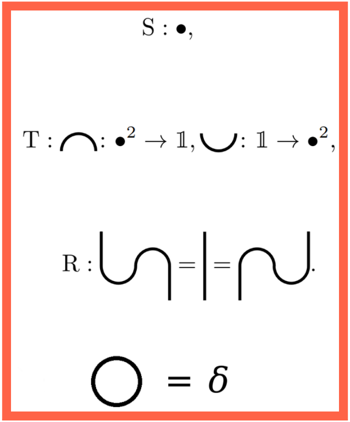
- Objects are just boundary sizes: $0, 1, 2, 3, \ldots$
- Morphisms are diagrams going from one boundary size to another
- Algebras The old algebra is the special case of diagrams from n to n

Two ways to compose pictures



- Vertical Stack one diagram on top of another, when the sizes match
- Horizontal Put diagrams side by side to build a wider diagram
- Machine These two moves are the basic grammar of diagram categories

A tiny example



- Cap A cap can be a morphism from two points to no points
- Cup A cup can be a morphism from no points to two points
- Loop Compose them one way and a closed circle appears

Why the category viewpoint helps

Temperley–Lieb algebra $TL_n(\delta)$

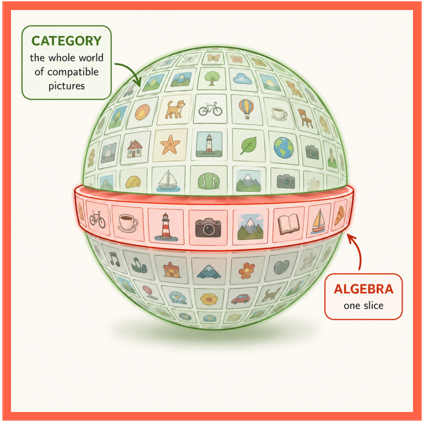
Fix a commutative ring R and $\delta \in R$.
$TL_n(\delta)$ is generated by $e_1, \ldots, e_{n-1}$, subject to:

$e_i^2 = \delta e_i$	$1 \le i \le n-1$
$e_i e_{i+1} e_i = e_i$	$1 \le i \le n-2$
$e_i e_{i-1} e_i = e_i$	$2 \le i \le n-1$
$e_i e_j = e_j e_i$	$\lvert i-j \rvert \ne 1$

same algebra, now described by generators and relations

- **Uniformity** One set of rules controls diagrams of every boundary size
- **Local pieces** Big pictures are built from small cups, caps, lines, and crossings
- **Crucial** The (monoidal) category has a more efficient presentation

What to remember from categories



- Category is the whole world of compatible pictures
- Algebra is one slice: diagrams from n points back to n points
- Next Change the allowed pictures and you enter the diagram zoo

Thank you for your attention!

I hope that was of some help.