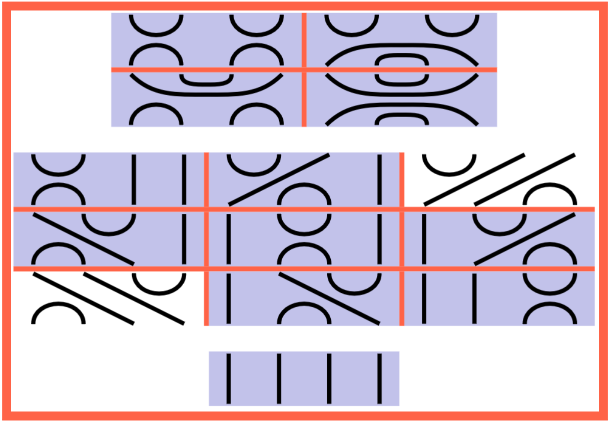
Diagram algebras - part 2

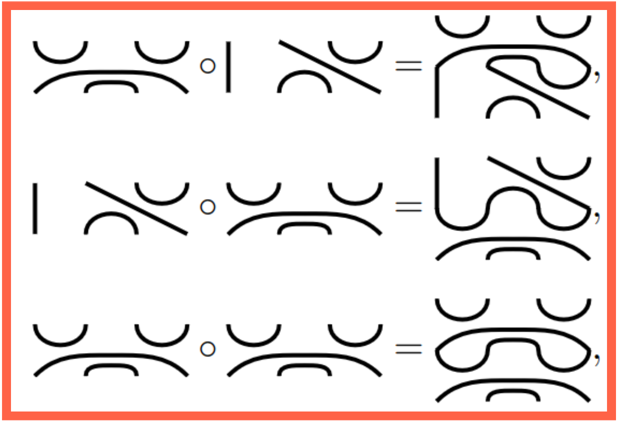
Or: Temperley–Lieb, the planar baby example

Temperley–Lieb in one sentence



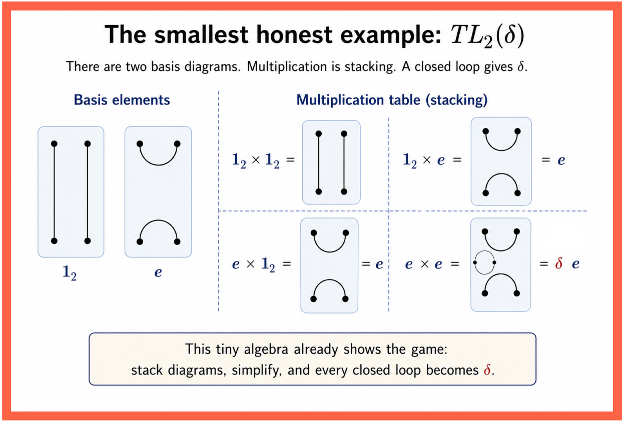
- Basis elements are non-crossing pairings of boundary points
- Multiplication means stack two pictures and clean up the middle
- Example A cup meeting a cap can close into a loop

The local rules



- Planarity Strands may bend, but they are not allowed to cross
- Loops Closed circles disappear and leave one loop weight δ
- Picture rule Multiply first, then replace every closed loop by a scalar

The smallest honest examples



- **Two points** There is essentially just one strand joining top to bottom
- **Four points** Now cups, caps, and through-strands can interact
- **First table** The multiplication table is small enough to draw by hand

Why this baby example matters

The same five non-crossing pairings, two lives

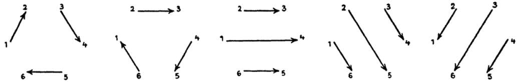
Temperley--Lieb basis diagrams

The five basis elements of $TL_3(\delta)$ are the following:

From Wikipedia

Rumer--Teller--Weyl valence-bond diagrams

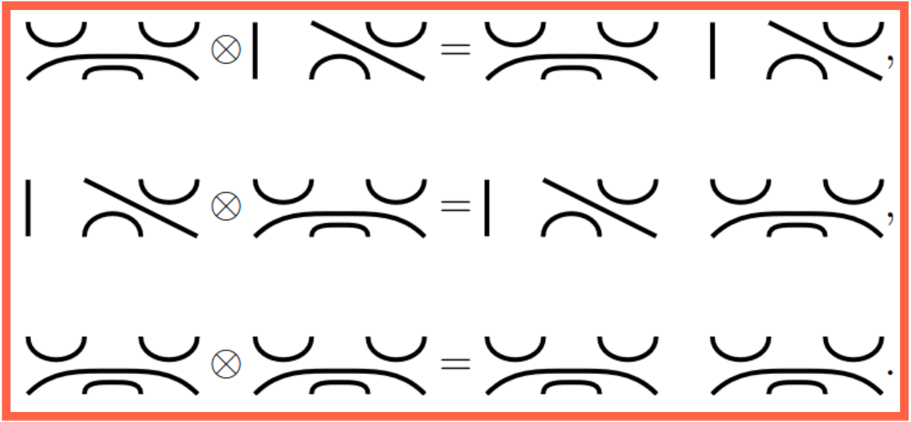
3. Als zweites Beispiel behandeln wir die zyklische einvalentige Kette mit sechs Atomen. Die Valenzfunktionen sind



From Rumer

- 1930s Rumer(–Teller–Weyl) already had these pictures (Invariant theory)
- 1970s Temperley–Lieb met them again (Statistical mechanics)
- 1980s Jones brought the same planar calculus to knots (Knot theory)

What to remember from TL



- Pictures TL diagrams are planar pairings, nothing crosses
- Computation Stacking produces loops, and loops become weights
- Next The category viewpoint turns this example into a whole machine

Thank you for your attention!

I hope that was of some help.