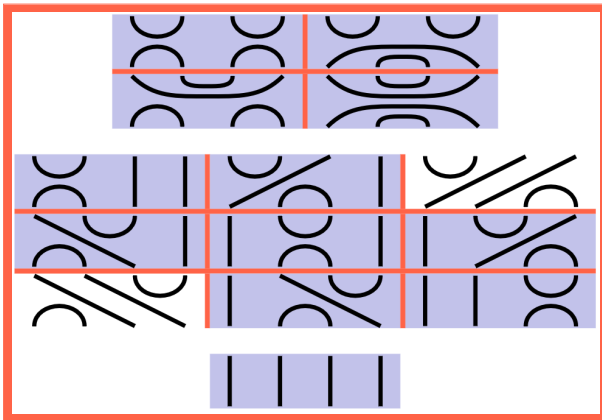


Diagram algebras - part 2

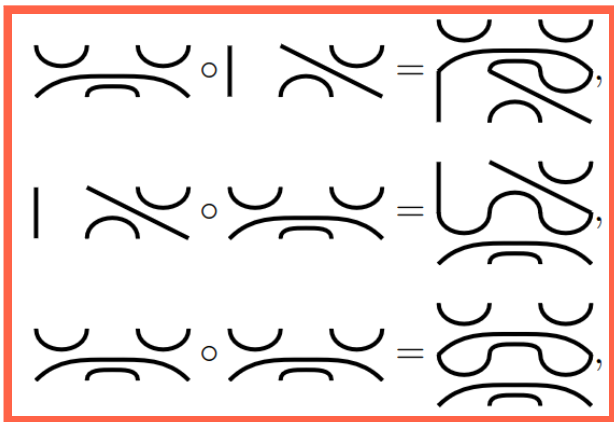
Or: Temperley–Lieb, the planar baby example

Temperley–Lieb in one sentence



-
- ▶ **Basis elements** are non-crossing pairings of boundary points
 - ▶ **Multiplication** means stack two pictures and clean up the middle
 - ▶ **Example** A cup meeting a cap can close into a loop

The local rules



- **Planarity** Strands may bend, but they are not allowed to cross
- **Loops** Closed circles disappear and leave one loop weight δ
- **Picture rule** Multiply first, then replace every closed loop by a scalar

The smallest honest examples

The smallest honest example: $TL_2(\delta)$

There are two basis diagrams. Multiplication is stacking. A closed loop gives δ .

Basis elements



Multiplication table (stacking)

$$1_2 \times 1_2 =$$



$$1_2 \times e =$$



$$= e$$

$$e \times 1_2 =$$



$$= e$$

$$e \times e =$$



$$= \delta e$$

This tiny algebra already shows the game:
stack diagrams, simplify, and every closed loop becomes δ .

- ▶ **Two points** There is essentially just one strand joining top to bottom
- ▶ **Four points** Now cups, caps, and through-strands can interact
- ▶ **First table** The multiplication table is small enough to draw by hand

Why this baby example matters

The same five non-crossing pairings, two lives

Temperley-Lieb basis diagrams

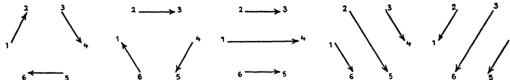
The five basis elements of $TL_3(\delta)$ are the following:



From Wikipedia

Rumer-Teller-Weyl valence-bond diagrams

3. Als zweites Beispiel behandeln wir die zyklische einvalentige Kette mit sechs Atomen. Die Valenzfunktionen sind



From Rumer

- ▶ 1930s Rumer(-Teller-Weyl) already had these pictures (Invariant theory)
- ▶ 1970s Temperley-Lieb met them again (Statistical mechanics)
- ▶ 1980s Jones brought the same planar calculus to knots (Knot theory)

What to remember from TL

The image displays three rows of string diagrams, each representing an equation in the Temperley-Lieb algebra. The diagrams are enclosed in a red rectangular border.

- Row 1:** A diagram consisting of two parallel horizontal arcs (top and bottom) with a vertical line segment crossing them, multiplied by the identity element (a vertical line with a small circle at the top and bottom), equals the same diagram as the identity element multiplied by the same diagram. The identity element is represented by a vertical line with a small circle at the top and bottom.
- Row 2:** A diagram consisting of two parallel horizontal arcs (top and bottom) with a vertical line segment crossing them, multiplied by the identity element, equals the same diagram as the identity element multiplied by the same diagram. The identity element is represented by a vertical line with a small circle at the top and bottom.
- Row 3:** A diagram consisting of two parallel horizontal arcs (top and bottom) with a vertical line segment crossing them, multiplied by the identity element, equals the same diagram as the identity element multiplied by the same diagram. The identity element is represented by a vertical line with a small circle at the top and bottom.

- **Pictures** TL diagrams are planar pairings, nothing crosses
- **Computation** Stacking produces loops, and loops become weights
- **Next** The category viewpoint turns this example into a whole machine

Thank you for your attention!

I hope that was of some help.