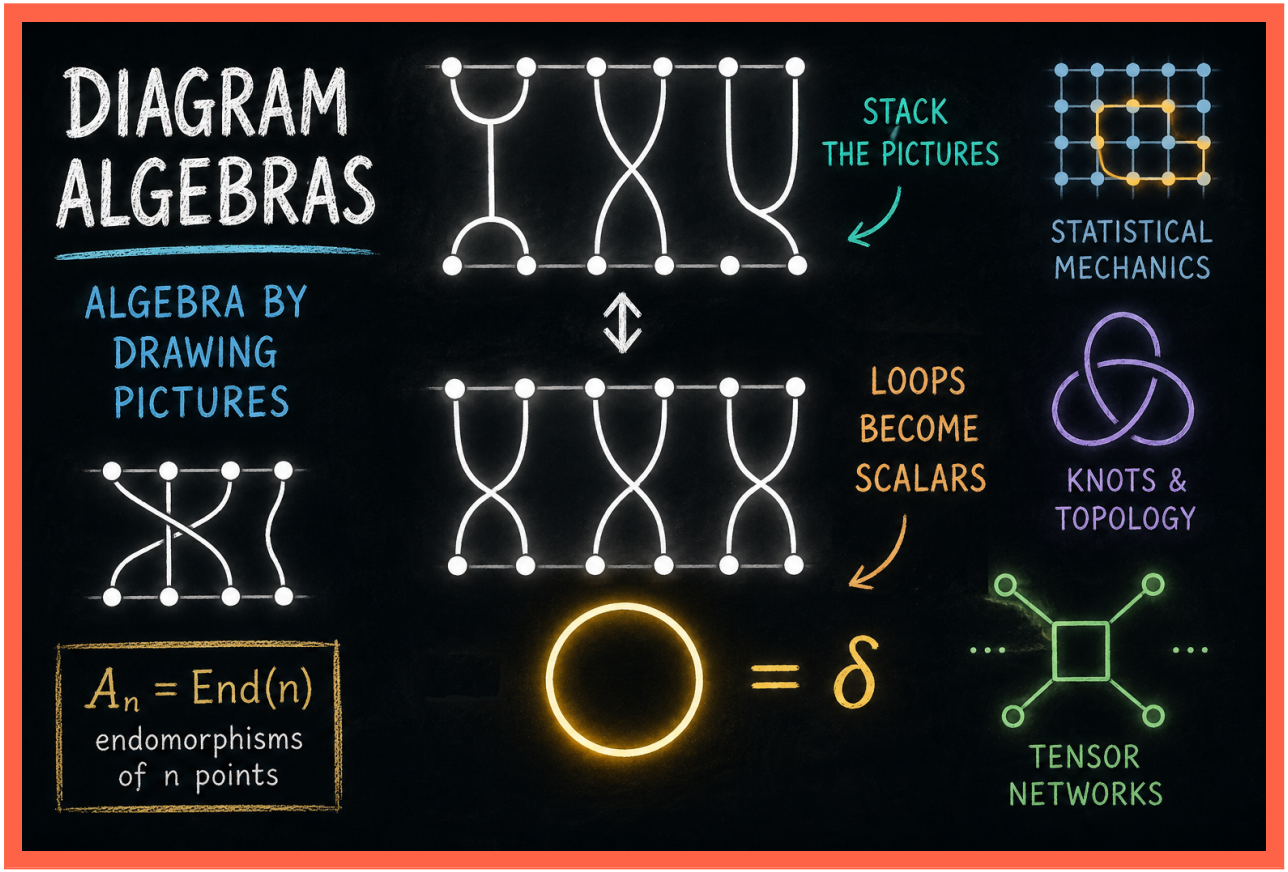
Diagram algebras - part 1?

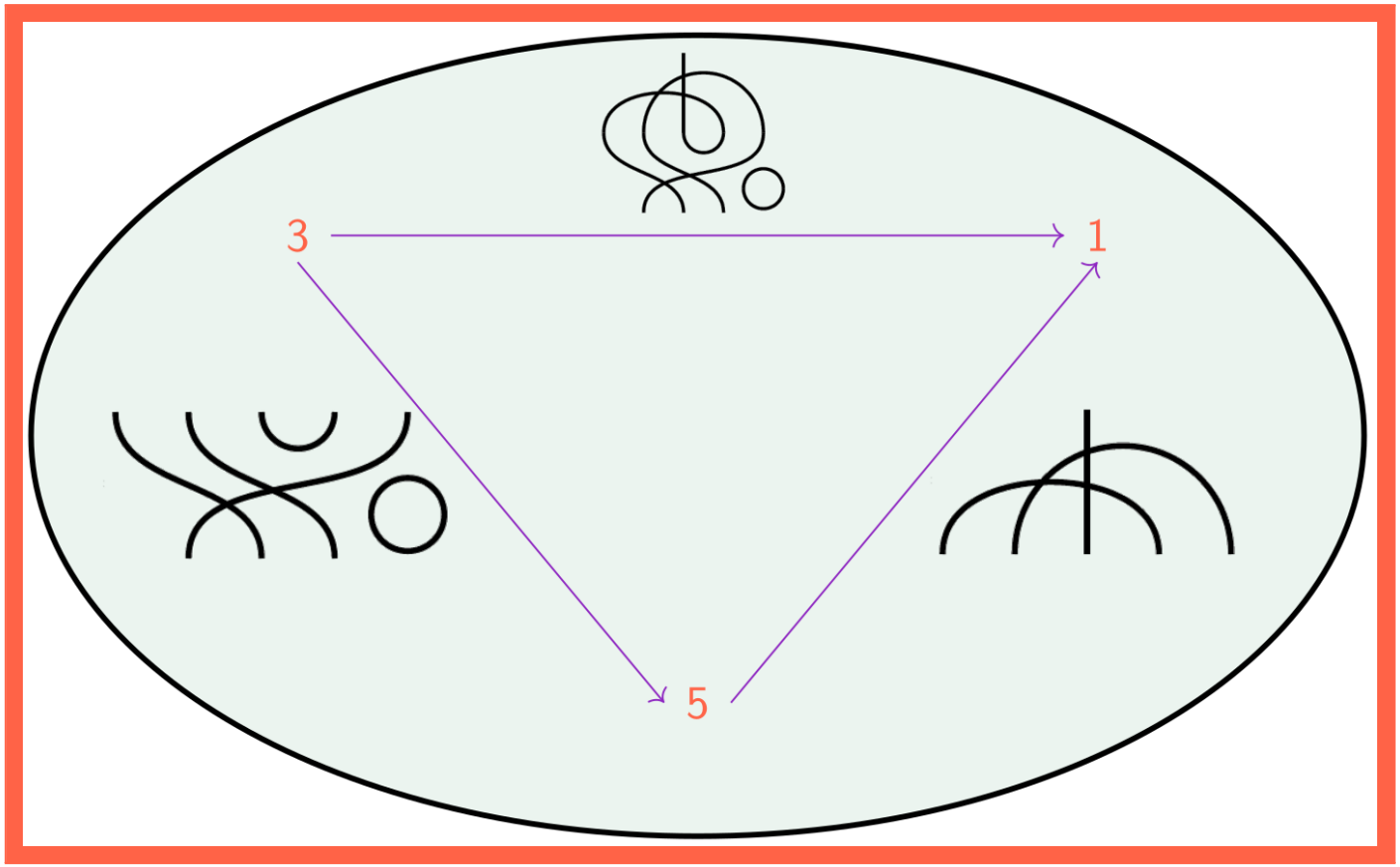
Or: Algebra by drawing pictures

What is a diagram algebra?



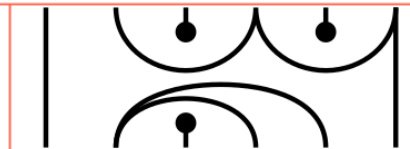
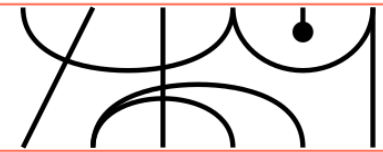
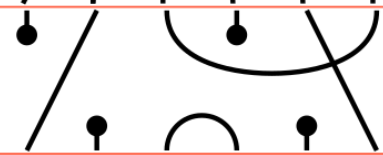
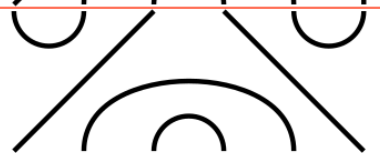
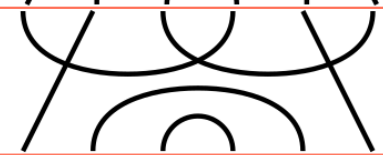
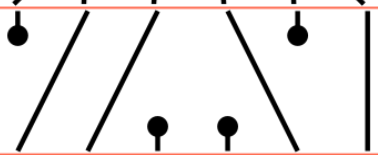
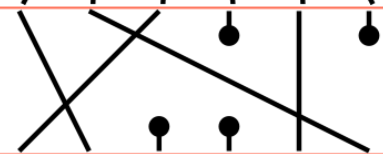
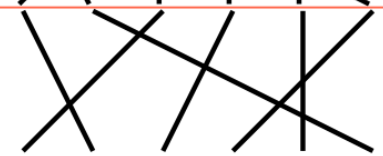
- Basis elements are pictures, not polynomials or matrices
- Multiplication means stack diagrams and simplify the middle
- Slogan Pictures compute: loops become scalars, crossings remember symmetry

Diagram categories come first



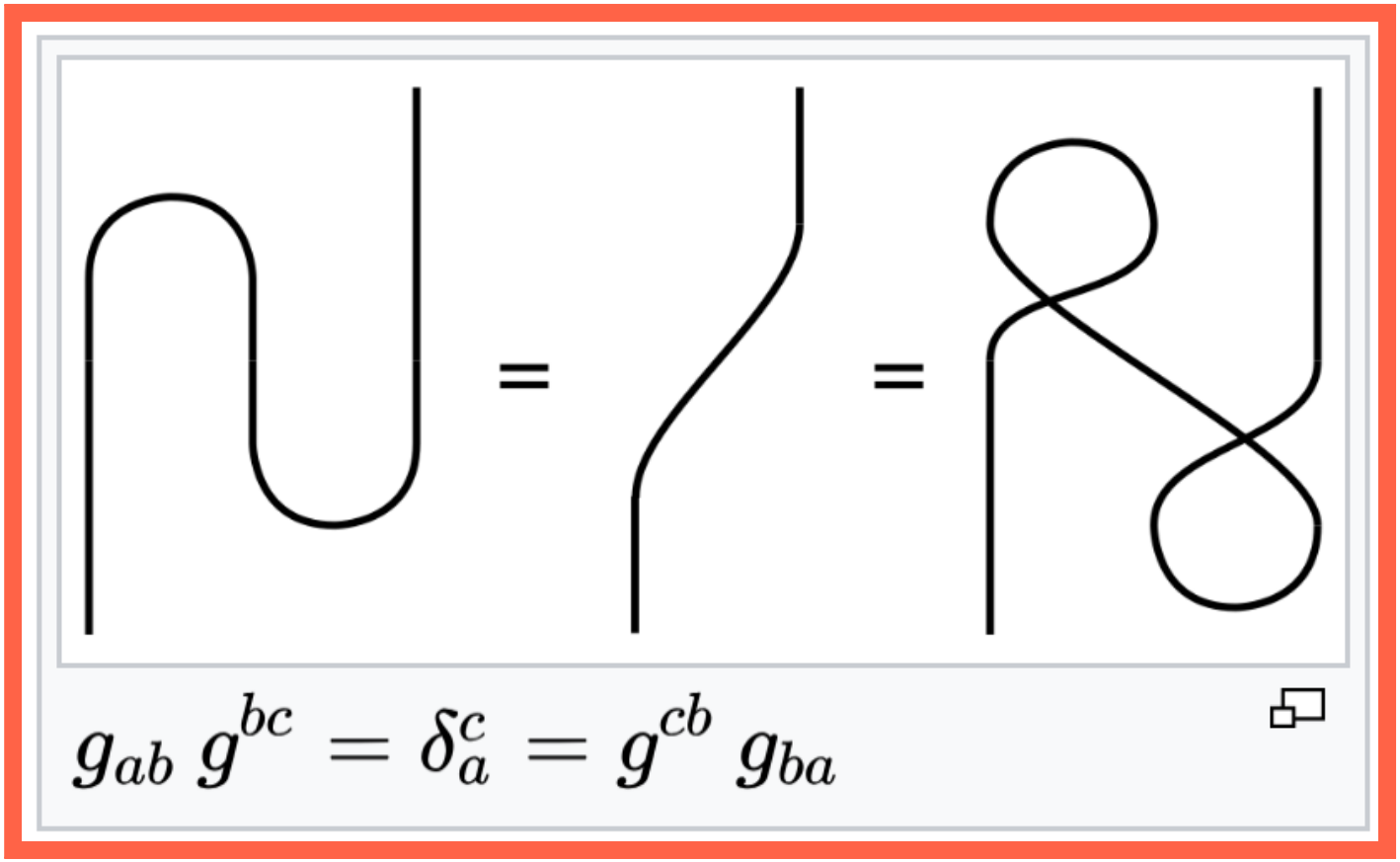
- Objects are usually numbers of boundary points
- Two operations vertical stacking = composition, horizontal stacking = tensor product
- Algebras appear as endomorphism algebras: $A_n = \mathrm{End}(n)$

The zoo: change the allowed pictures

Symbol	Diagrams	Big reps	Symbol	Diagrams	Big reps
$\mathrm{p}\mathcal{P}\mathrm{a}_n$		YES*	$\mathcal{P}\mathrm{a}_n$		YES_c^*
$\mathcal{M}\mathrm{o}_n$		YES_c	$\mathcal{R}\mathrm{o}\mathcal{B}\mathrm{r}_n$		YES_c^*
$\mathcal{T}\mathcal{L}_n$		YES	$\mathcal{B}\mathrm{r}_n$		YES*
$\mathrm{p}\mathcal{R}\mathrm{o}_n$		YES*	$\mathcal{R}\mathrm{o}_n$		YES*
$\mathrm{p}\mathcal{S}_n$		EX	$\mathcal{S}_n$		NO

- Planar Temperley–Lieb (TL): no crossings, only pairings
- Crossings Brauer: pairings with permutation-like middle pieces
- Bigger components Partition, rook, Motzkin, blob, affine, decorated variants

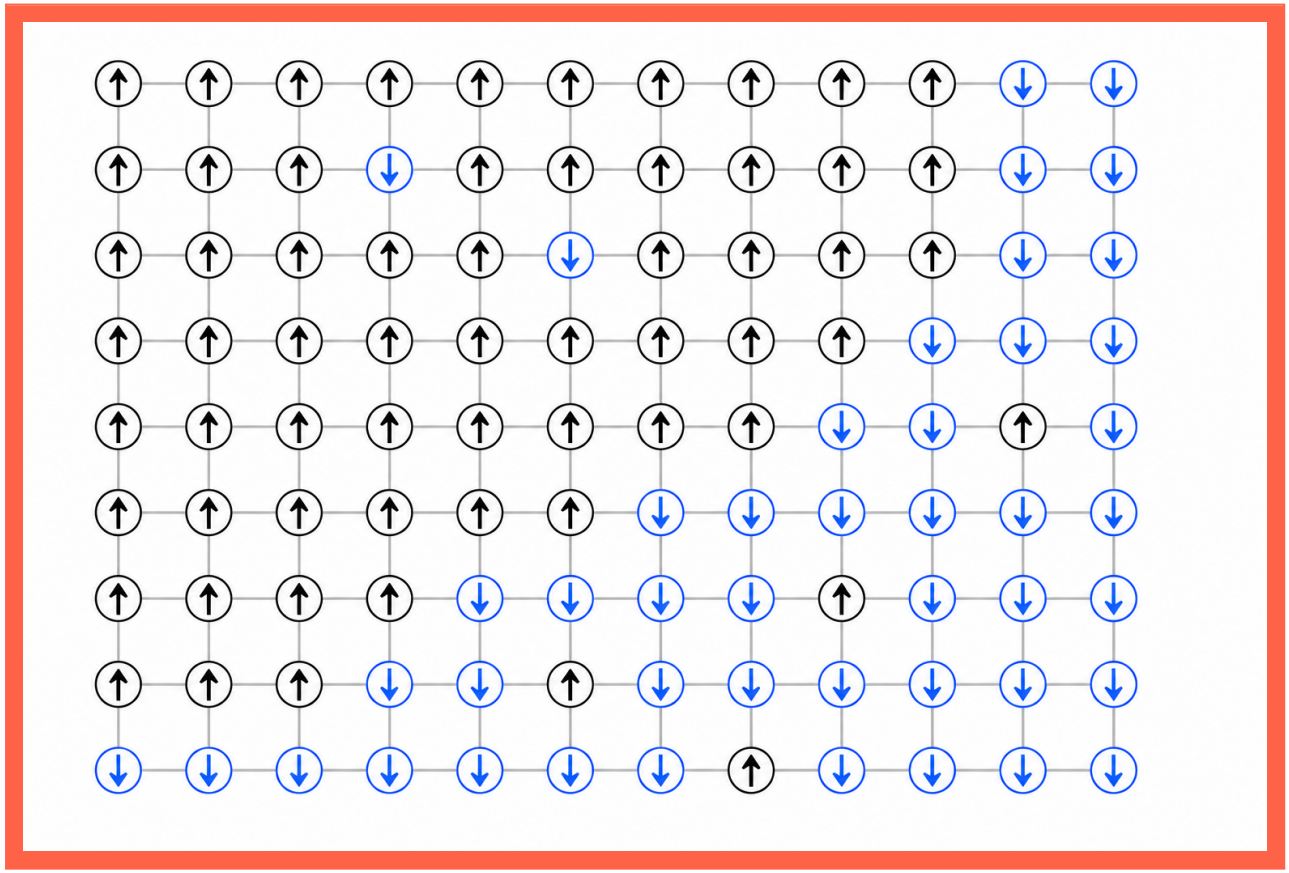
Why care? Appearances outside algebra



$$g_{ab}\, g^{bc} = \delta_a^c = g^{cb}\, g_{ba}$$

- **Statistical mechanics** Transfer matrices, loop models, Potts/XXZ flavor
- **Topology** Kauffman bracket, Jones polynomial, subfactor diagrams
- **Tensor calculus** Networks, traces, invariant tensors, graphical computations

Roadmap: where the series is going



- A Stacking pictures: TL, Brauer, partition, rook, Motzkin, blob
- B Applications: statistical mechanics, knots, tensor networks, invariants
- C Structure: centralizers, sandwiches, semisimplicity, growth

Thank you for your attention!

I hope that was of some help.