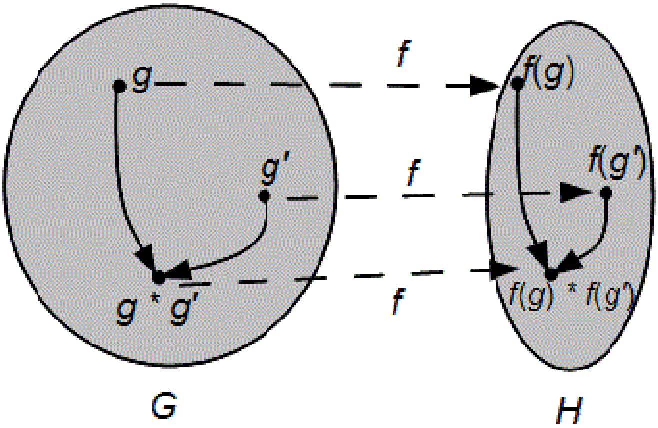
What are...functors?

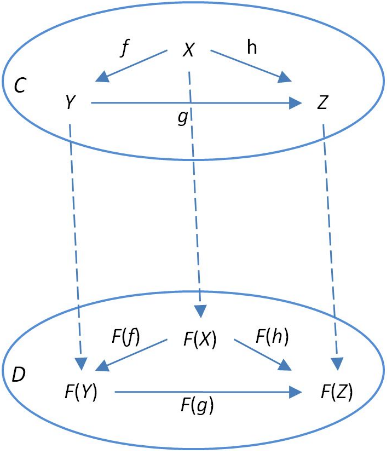
Or: Maps between categories

Preserving structure – classical



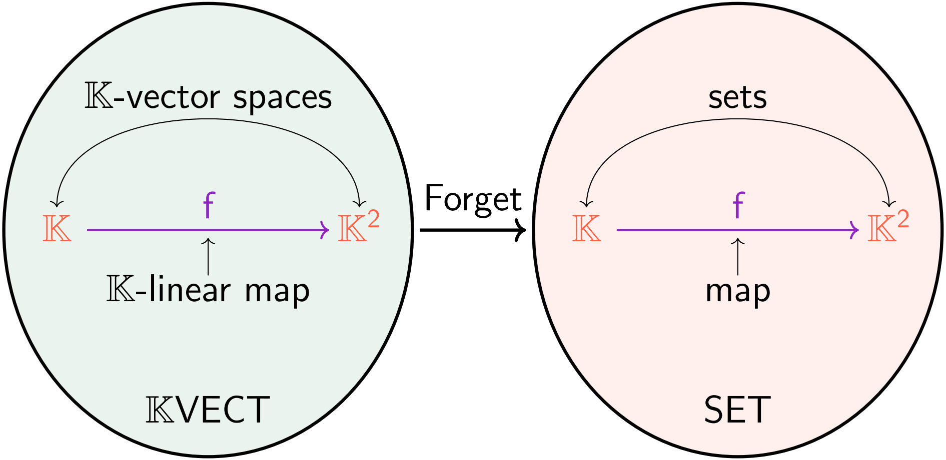
- A group homomorphism f is the correct map between groups
- Why? Because it preserves the only relevant structure $f(ab) = f(a)f(b)$
- Same for ring homomorphisms, $\mathbb{K}$-linear maps, and many more

Preserving structure – categorical



- A functor F should be the correct map between categories
- F should associate objects to objects $X \mapsto F(X)$
- F should associate arrows to arrows $f \mapsto F(f)$ such that $F(gf) = F(g)F(f)$

Forgetting is easy



- Forgetful functor A functor from a rich category to a lean category
- Example Forget: $\mathbb{K}$VECT $\to$ SET
- This functor forgets that X is a $\mathbb{K}$-vector space and the f is $\mathbb{K}$-linear

For completeness: A formal definition

A **functor** F from C to D is a mapping that:

- associates each object X in C to an object $F(X)$ in D
- associates each arrow $f\colon X \to Y$ in C to an arrow $F(f)\colon F(X) \to F(Y)$ in D

such that:

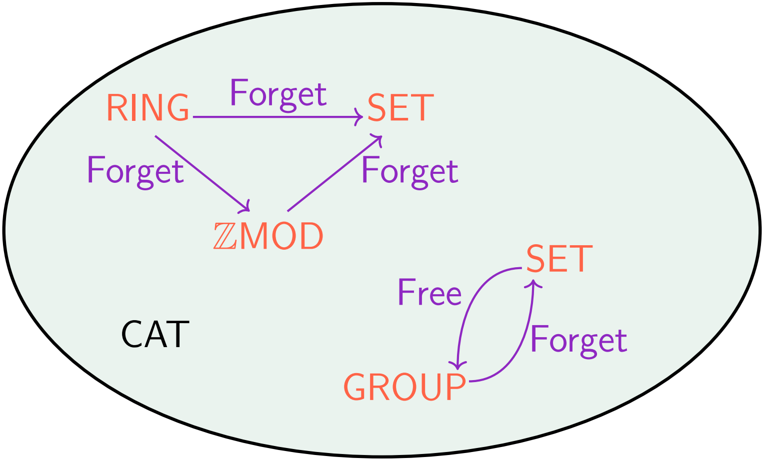
(a) $F(id_X) = id_{F(X)}$ "Unit goes to unit"

(b) $F(gf) = F(g)F(f)$ Composition is preserved

As usual:

- There is an identity functor $id_C\colon C \to C$
- Compositions of functors are functors
- Thus, $Fun(C, C)$ is a monoid
- Actually, $Fun(C, D)$ is a category but this will have to wait for a while

Category theory takes itself serious



- Categories themselves form a category CAT where arrows are functors
- Well, almost There are set-theoretical issues with CAT, but let us ignore that

Thank you for your attention!

I hope that was of some help.