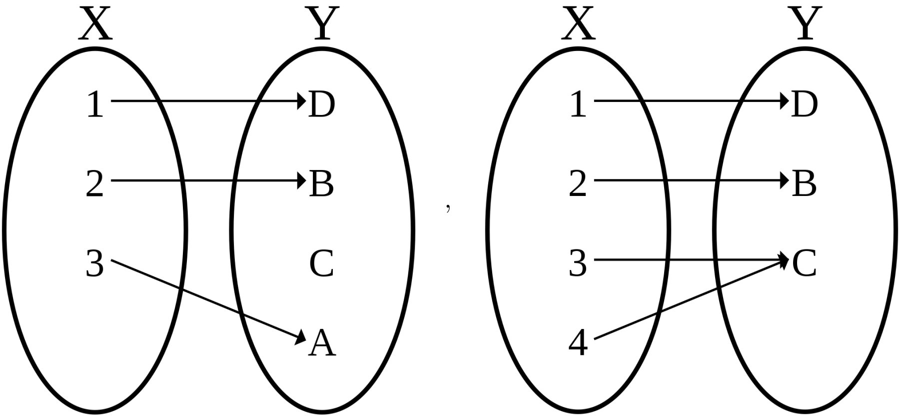
What is...monic-epic-iso?

Or: Not quite injective-surjective-bijective

Injectives map

f injective | g not injective



- A map f between sets is injective $\Leftrightarrow$ ($f(x) = f(y)$ implies $x = y$)
- This description needs elements Bad for category theory
- Task We need a element-free description

Injective maps – revisited

$$
\begin{tikzcd}
Z \arrow[r, shift left, "g_1"] \arrow[r, shift right, "g_2"'] & X \arrow[r, "f"] & Y
\end{tikzcd}
$$

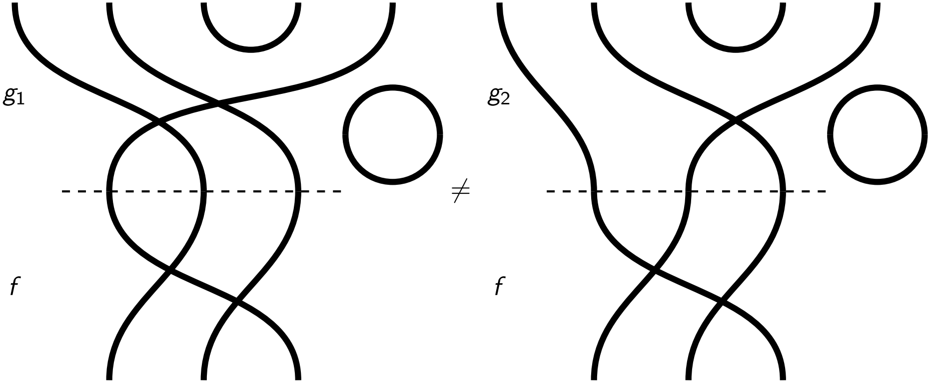
- A map f between sets is injective $\Leftrightarrow$ ($fg_1 = fg_2$ implies $g_1 = g_2$)
- This description needs no elements Good for category theory
- Example With f as on the previous slide

$$g_1\colon \begin{cases} 1 \mapsto 2, \\ 2 \mapsto 2, \end{cases} \quad g_2\colon \begin{cases} 1 \mapsto 2, \\ 2 \mapsto 1, \end{cases}$$

$$f\colon \begin{cases} 1 \mapsto D, \\ 2 \mapsto B, \\ 3 \mapsto A \end{cases}$$

$$fg_1(1) = B \neq D = fg_2(1) \quad \text{and} \quad g_1 \neq g_2$$

“Injective maps” in 1COB



$$g_1 \neq g_2$$

- 1COB ($fg_1 = fg_2$ implies $g_1 = g_2$) still makes sense
- This has nothing to do with the element definition of injective
- Above Note that 1-manifolds are abstract

For completeness: A formal definition

In an arbitrary category C:

- f is monic $\Leftrightarrow$ ($fg_1 = fg_2$ implies $g_1 = g_2$) "injective"

$$\begin{tikzcd} Z \arrow[r, shift left, "g_1"] \arrow[r, shift right, "g_2"'] & X \arrow[r, "f"] & Y \end{tikzcd}$$

- f is epic $\Leftrightarrow$ ($g_1 f = g_2 f$ implies $g_1 = g_2$) "surjective"

$$\begin{tikzcd} X \arrow[r, "f"] & Y \arrow[r, shift left, "g_1"] \arrow[r, shift right, "g_2"'] & Z \end{tikzcd}$$

- f is an iso $\Leftrightarrow$ ($\exists\ g(= f^{-1})$ with $gf = id_Y$ and $fg = id_Y$) "bijective"

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend right, "id_X"'] & Y \arrow[r, "g"] & X \end{tikzcd}, \quad \begin{tikzcd} Y \arrow[r, "g"] \arrow[rr, bend right, "id_Y"'] & X \arrow[r, "f"] & Y \end{tikzcd}$$

Beware: these are honest generalizations

$$C = \begin{tikzcd} 1 \arrow[rr, "f"] \arrow[dr, "gf"'] & & 2 \arrow[dl, "g"] \\ & 3 & \end{tikzcd}$$

- In most set-based categories
  - monic=injective
  - epic=surjective
  - iso=bijective
- In C above
  - monic=all non-identity arrows
  - epic=all non-identity arrows
  - iso=no arrow

Thank you for your attention!

I hope that was of some help.