What are...Kan extensions?

Or: Recover information; kind of...

Databases

$$C = \quad People1 \xrightarrow{dating} People2 \;, \qquad D = \quad People1$$

$$G : D \to C,\ G(People1) = People1$$

$F : C \to \mathrm{SET} \rightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

Adam	Fabio
Babhru	Jun
Claus	Fabio
Deepti	Eve

People2
Eve
Fabio
Gwenyth
Hamza
Inez
Jun

$D \to \mathrm{SET} \rightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

- Functors $C \to \mathrm{SET}$ and $D \to \mathrm{SET}$ are like a database
- $\_ \circ G : [C, \mathrm{SET}] \to [D, \mathrm{SET}]$ forgets all about dating Easy

Left is generous

$$C = \quad People1 \xrightarrow{dating} People2 \;, \qquad D = \quad People1$$

$$G\colon D \to C, G(People1) = People1$$

$Lan_G F\colon C \to \mathrm{SET} \leftrightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

Adam	Person1
Babhru	Person2
Claus	Person3
Deepti	Person4

People2
Person1
Person2
Person3
Person4

$F\colon D \to \mathrm{SET} \leftrightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

- Recovering lost data can not work without cost Hard
- The left Kan extension $Lan_G F$ tries to recover the data generously

Right is conservative

$$C = \quad People1 \xrightarrow{dating} People2 \ , \quad D = \quad People1$$

$$G\colon D \to C, G(People1) = People1$$

$Ran_G F\colon C \to \mathrm{SET} \rightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

Adam	Person1
Babhru	Person1
Claus	Person1
Deepti	Person1

People2
Person1

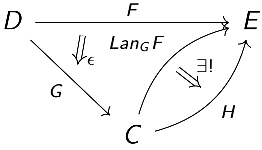
$F\colon D \to \mathrm{SET} \rightsquigarrow$

People1
Adam
Babhru
Claus
Deepti

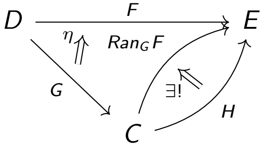
- Recovering lost data can not work without cost Hard
- The right Kan extension $Ran_G F$ tries to recover the data conservatively

For completeness: A formal definition

A left Kan extension of $F\colon D \to E$ along $G\colon D \to C$ is given by a functor $Lan_G\colon C \to E$ and a nat trafo $\epsilon\colon F \Rightarrow (Lan_G F)G$, and the universal diagram



A right Kan extension of $F\colon D \to E$ along $G\colon D \to C$ is given by a functor $Ran_G\colon C \to E$ and a nat trafo $\eta\colon (Ran_G F)G \Rightarrow F$, and the universal diagram



- These might not exists
- If they exist, then they are unique up to unique isomorphism

Kan extensions everywhere

- The limit of $F\colon D \to E$ is $Ran_G F(\bullet)$ for $G\colon D \to \bullet$
- The colimit of $F\colon D \to E$ is $Lan_G F(\bullet)$ for $G\colon D \to \bullet$
- $Ran_G id_D$ is the left adjoint of $G\colon D \to C$
- $Lan_G id_D$ is the right adjoint of $G\colon D \to C$

Thank you for your attention!

I hope that was of some help.