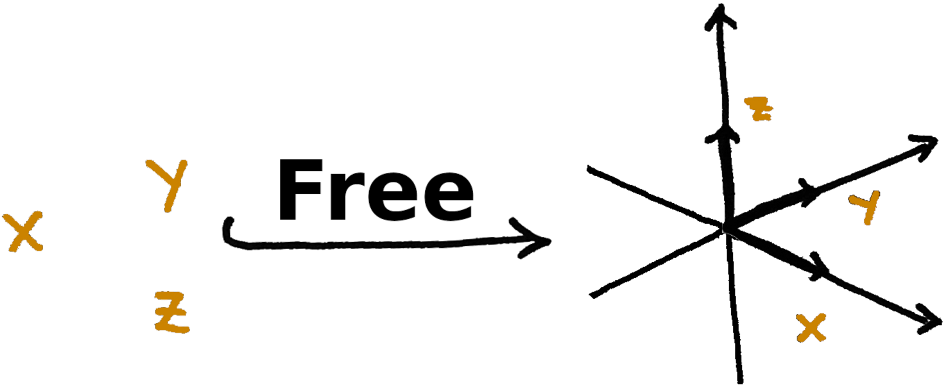
What are...examples of adjoint functors?

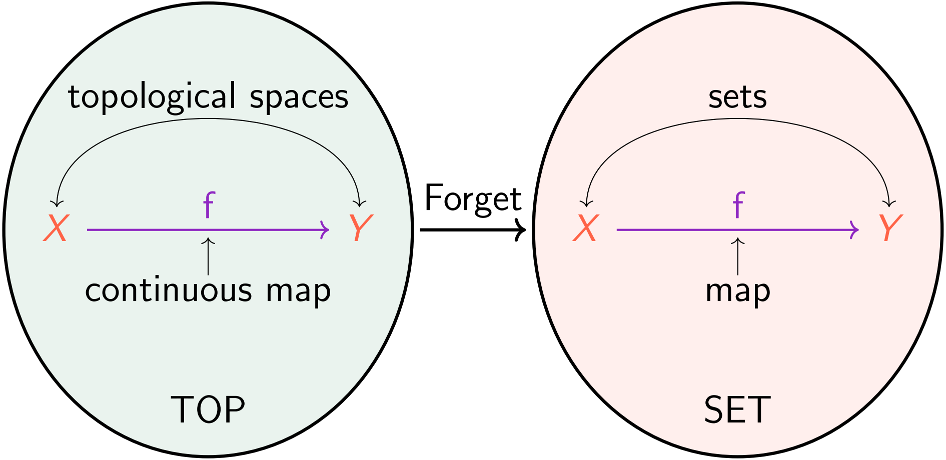
Or: Adjoints occur almost everywhere

Free and forget



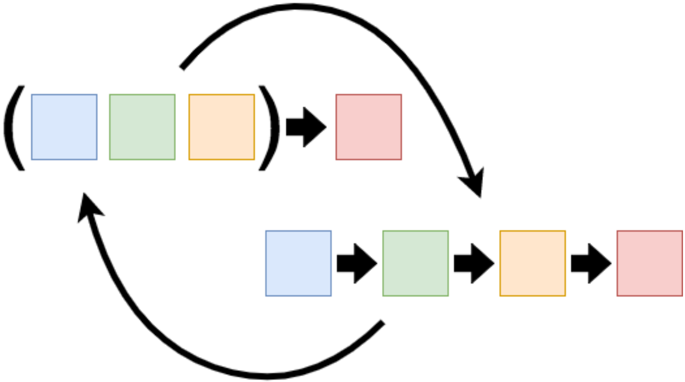
- The $(\mathit{Free}, \mathit{Forget})$ adjunction appears very often
- Informally "Free = generic", satisfying only the necessary relations
- Example Free vector spaces Non-example Free fields do not exist

Free and cofree



- Forget:TOP→SET has two adjoint functors
- The left adjoint uses the discrete topology Free topology
- The right adjoint uses the indiscrete topology Cofree topology

Currying in Math and CS



- The endofunctors $\left(\_ \times Y, \hom_{\mathrm{SET}}(\_, Y)\right)$ on SET are an adjoint pair:

$$\boxed{\hom_{\mathrm{SET}}(X \times Y, Z) \cong \hom_{\mathrm{SET}}\left(X, \hom_{\mathrm{SET}}(Y, Z)\right)}$$

  Converts a multiple input map into maps where each takes a single argument

- The isomorphisms are known as (un)currying

For completeness: A list

Here is a list of important adjoint functors (F, G) with $G\colon X \rightleftarrows Y\colon F$

- The free-forget adjunction (F=Forget, G=Free)

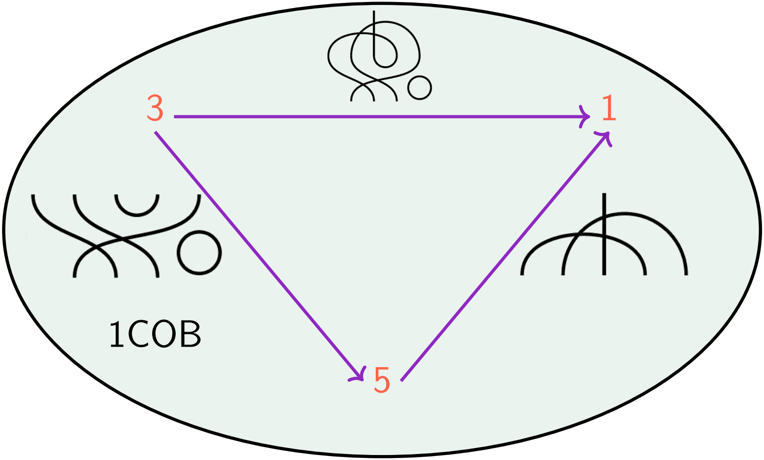
X	$\mathbb{K}$VECT	MONOID	GROUP	$\mathbb{Z}$MOD	RING	$\mathbb{K}$ALG	CAT
Y	SET	SET	SET	SET	SET	SET	QUIVER

- X=FIELD, Y=DOMAIN, F=Forget, G=Field of fractions
- X=$\mathbb{C}$VECT, Y=$\mathbb{R}$VECT, F=Forget, G=Scalar extension one can vary $\mathbb{R}$, $\mathbb{C}$
- X=pRING, Y=RING, F=Forget, G=Polynomial ring
- The tensor-hom adjunction, *e.g.* currying

$$\mathrm{hom}_C(X \otimes_D Y, Z) \cong \mathrm{hom}_D\big(X, \mathrm{hom}_C(Y, Z)\big)$$

- X=RING, Y=GROUP, F=Group of units, G=Group ring
- X=$\mathbb{Z}$MOD, Y=GROUP, F=Include, G=Abelianization
- Any equivalence is an adjoint pair
- Many more!

Back to cobordisms



- PIVSYM=Pivotal symmetric monoidal categories A mouthful ;-)
- Forget:PIVSYM→SET has a left adjoint, 1COB arises in this way Free!
- Warning There are some crucial nontrivial details which I ignore here

Thank you for your attention!

I hope that was of some help.