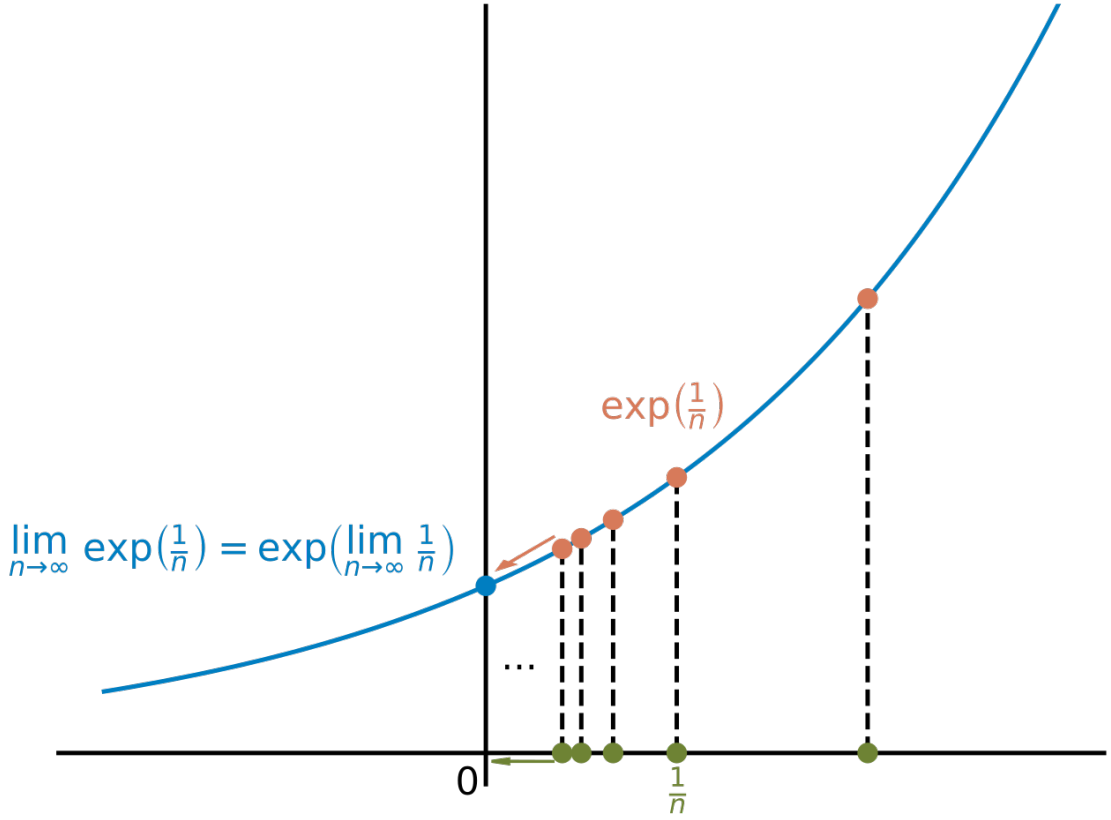
What are...continuous functors?

Or: Functors not functions

Continuous = preserving limits



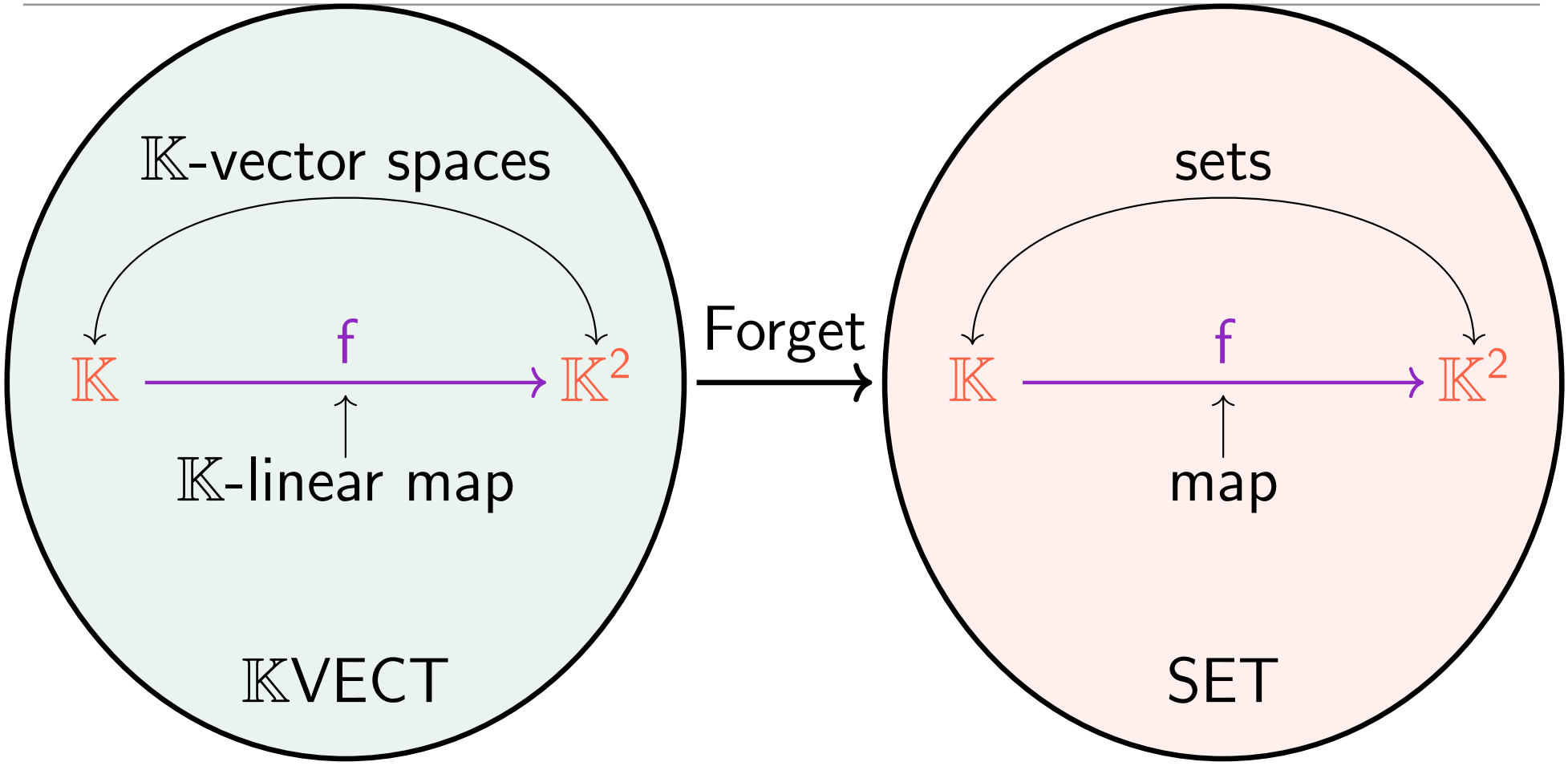
- A function is continuous if and only if (appropriately interpreted)

$$f(\lim_{n\to\infty} d_n) = \lim_{n\to\infty} f(d_n)$$

- A functor should be continuous if and only if (appropriately interpreted)

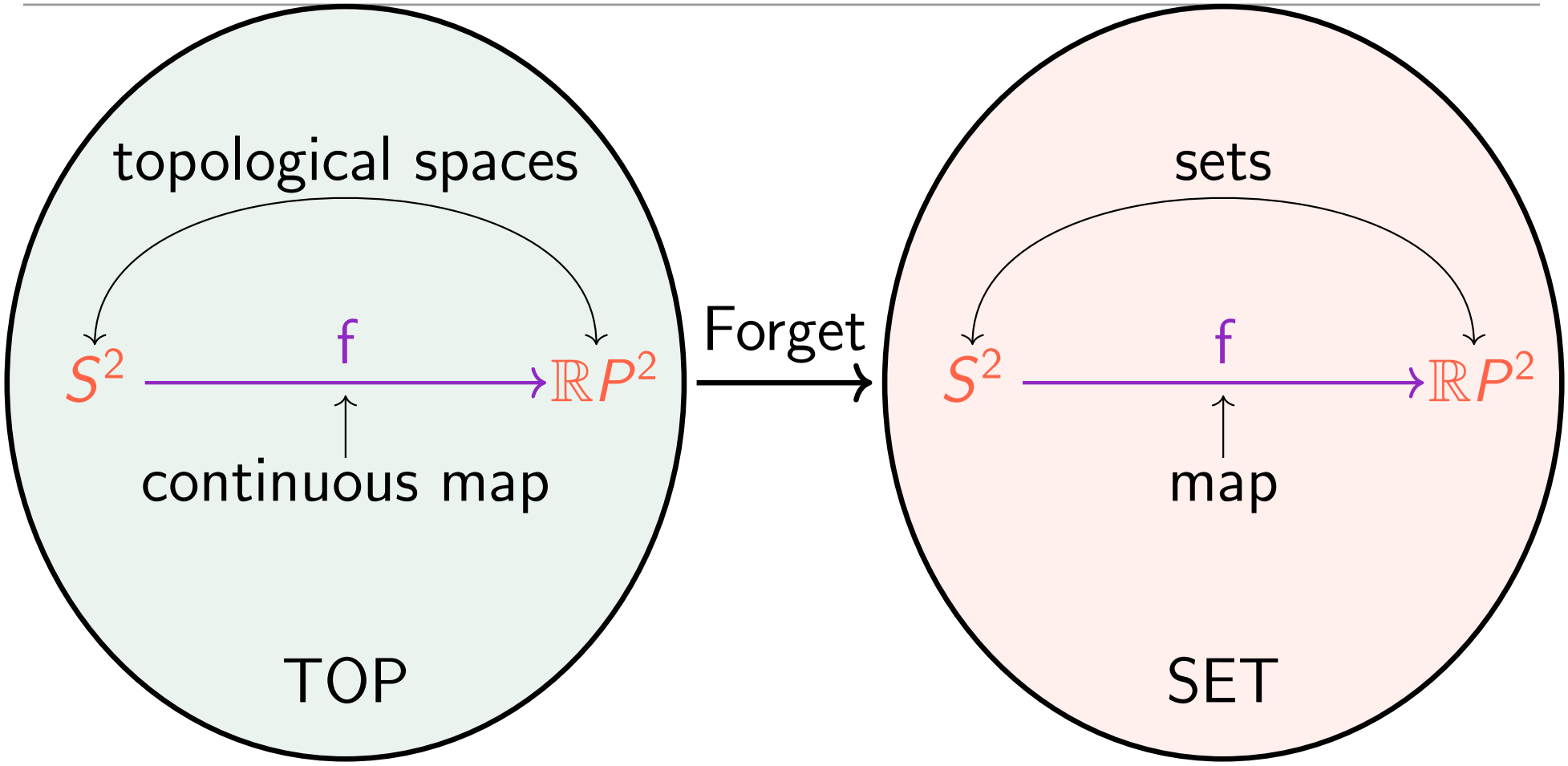
$$F(\lim \mathcal{D}) \cong \lim F\mathcal{D}$$

Don't forget vector spaces!



- $\oplus$ is a (finite) product and a coproduct in $\mathbb{K}$VECT
- Forget($\oplus$) is a product in SET
- Forget($\oplus$) is not a coproduct in SET

Forget topological spaces!



- $\prod / \coprod$ are product respectively coproduct in TOP
- Forget($\prod$) is a product in SET
- Forget($\coprod$) is a coproduct in SET

For completeness: A formal definition

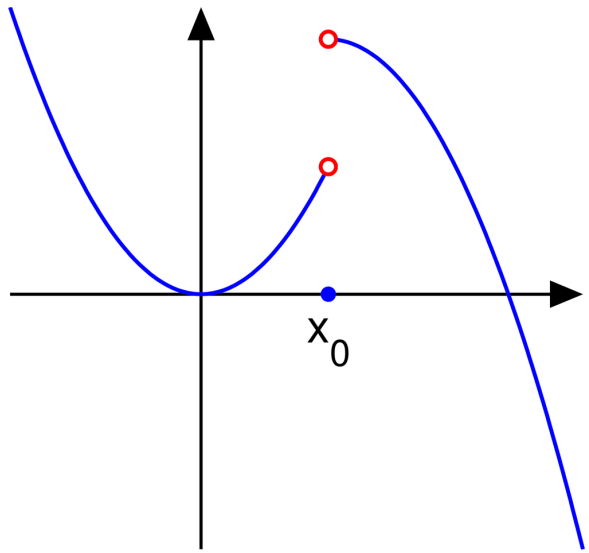
A functor $F : C \to D$ is...

- ... (finitely) continuous if preserves all (finite) limits
- ... (finitely) cocontinuous if preserves all (finite) (co)limits

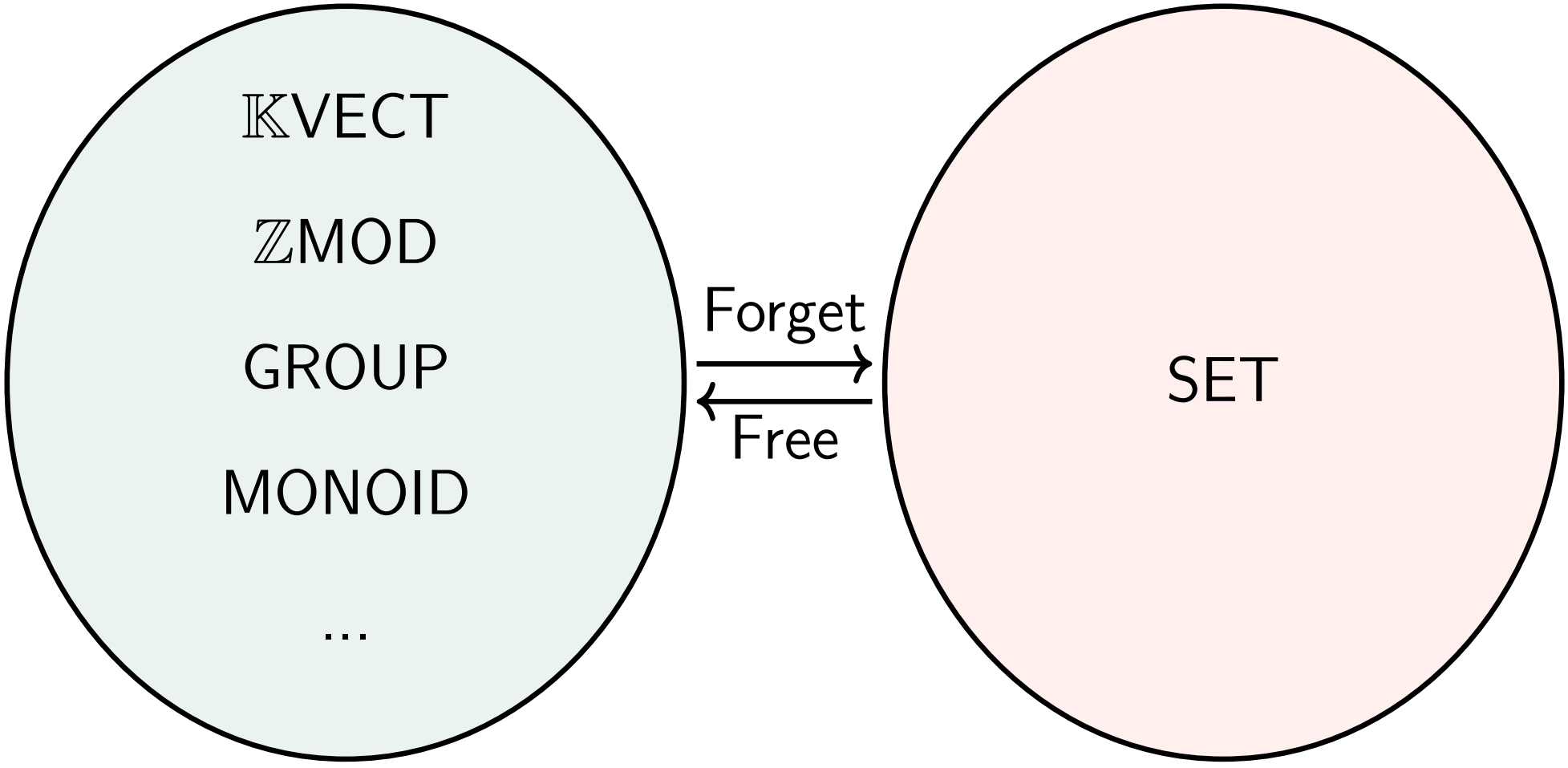
- The definition is mostly useful when C is (co)complete
- Write a limit $\mathcal{D} : I \to C$ as $\lim \mathcal{D}$, then

$$\text{Continuous: } F(\lim \mathcal{D}) = \lim \mathcal{F} \circ \mathcal{D} = \lim \mathcal{F}\mathcal{D}$$

- Partially continuous functors preserve *e.g.* products but not equalizer



Examples



- Hom functors $\hom_C(X, \_)$ are continuous
- Forgetful functors are often continuous but not cocontinuous
- Free functors are often cocontinuous but not continuous

Thank you for your attention!

I hope that was of some help.