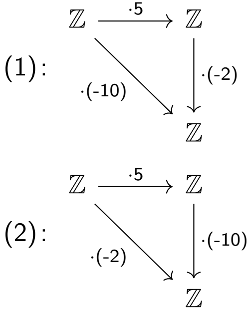
What are...(commuting) diagrams?

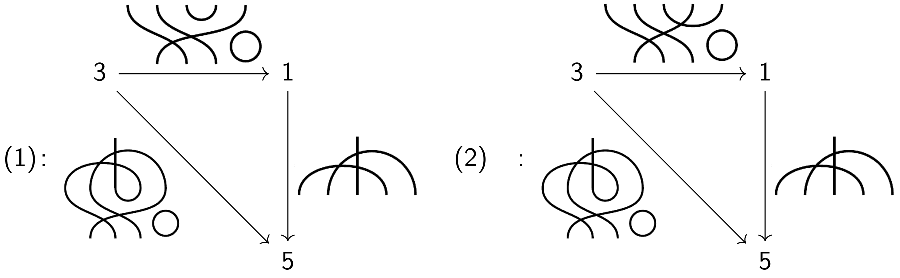
Or: Graphs and paths

Diagrams in SET



- (1) Going right+down equals going right-down
- (2) Going right+down does not equal going right-down
- We call (1) commutative
- We say that (2) does not commute

Diagrams in 1COB



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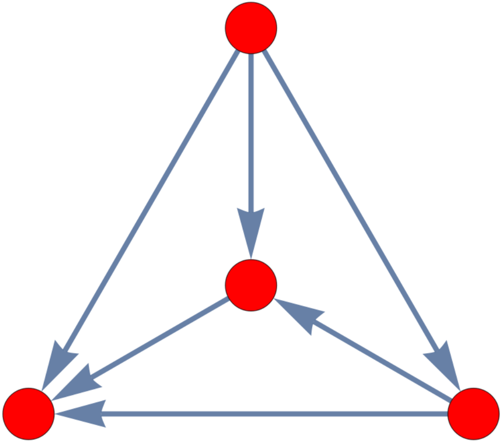
Paths



- An abstract diagram is a directed graph J
- We can interpret J in any category C
- J commutes in C if all paths with the same start and end commute in C

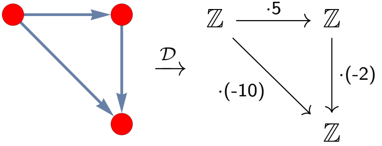
For completeness: A formal definition

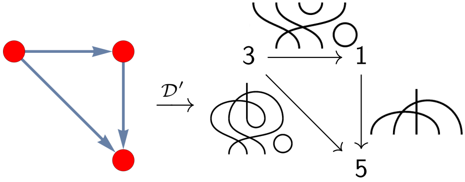
A diagram $\mathcal{D}$ of shape J in C is an association

$$\mathcal{D}\colon J \to C$$

It commutes if all directed paths in $\mathcal{D}(J)$ with the same start and endpoints lead to the same result in C

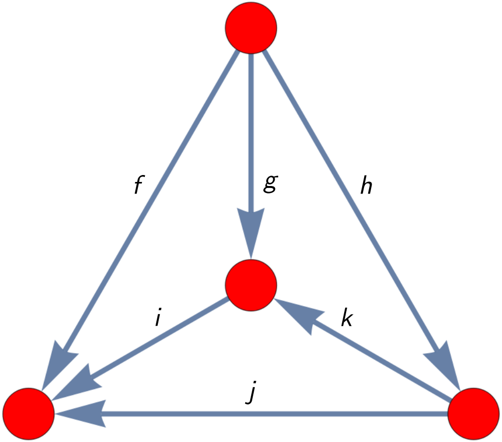
- One shape, many diagrams:





- "association" is replaced by functor as soon as that concept is introduced
- The actual objects and morphisms in J are largely irrelevant
- J commutes $\Rightarrow \mathcal{D}(J)$ commutes, but it can happen that $\not\Leftarrow$

Commuting faces



- Very often it suffices to check that faces commute
- Example $f = jh$ follows from $f = ig$, $g = kh$ and $j = ik$

Thank you for your attention!

I hope that was of some help.