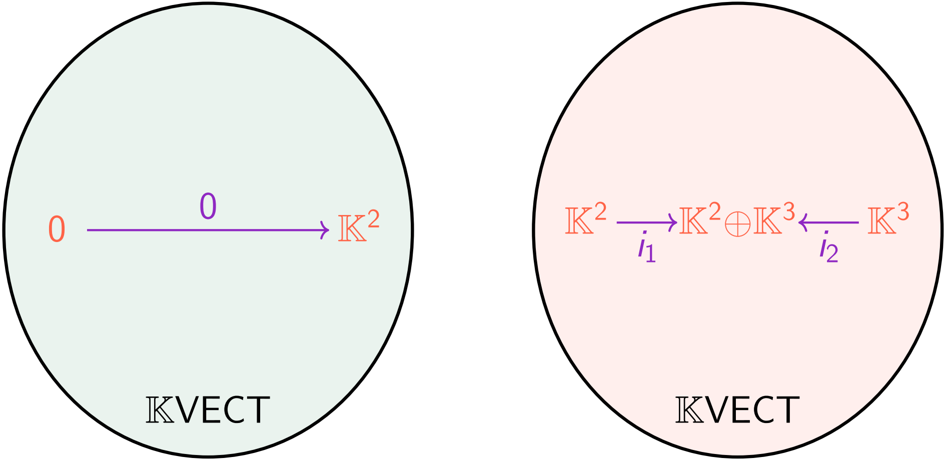
What are...(co)equalizers?

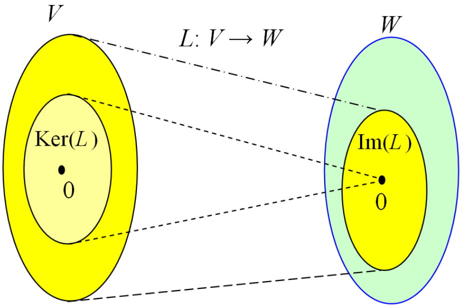
Or: Why kernels?

Vector spaces again



- $\mathbb{K}$VECT We have seen 0 (initial+terminal), $\oplus$ (product+coproduct)
- Also crucial: kernels $\ker(f)$+cokernels $\mathrm{coker}(f) = Y/\mathrm{im}(f)$
- What is the universal property of (co)kernels?

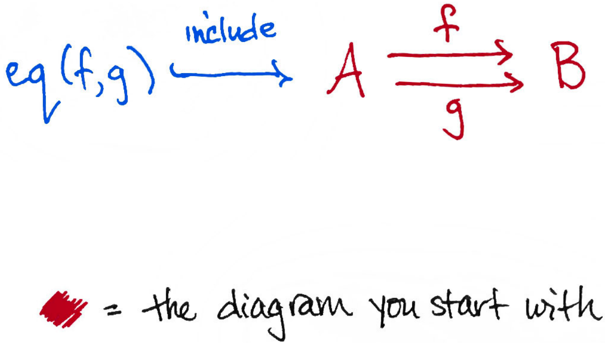
Kernel and image



- Problem The usual definition of kernels and images are set-based

$$\ker(L) = \{a \in V | L(a) = 0\} \quad \mathrm{im}(L) = \{b \in W | \exists a \in V : L(a) = b\}$$

- Task Get rid of sets!



- $\big(\ker(L), \iota\big)$ is the subspace of V such that $L \circ \iota = 0 \circ \iota$:

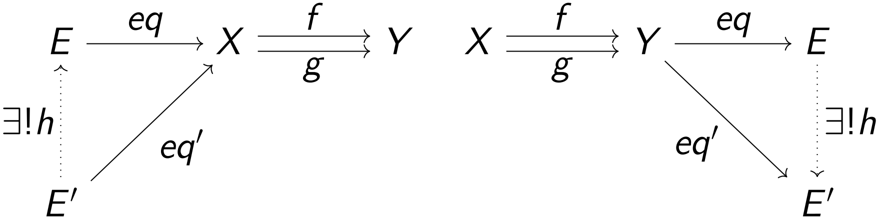
$$\ker(L) \xhookrightarrow{\ \mathit{incl.}\ } V \underset{0}{\overset{L}{\rightrightarrows}} W$$

- The cokernel can be described dually

For completeness: A formal definition

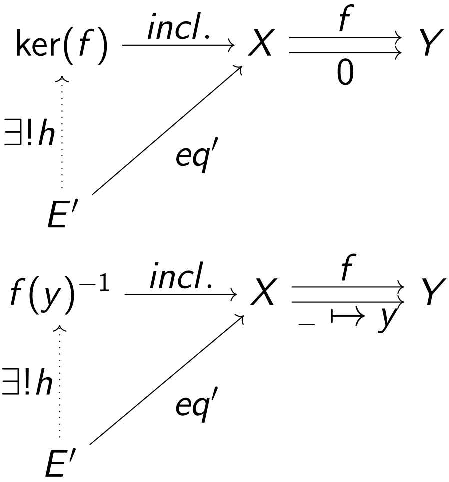
A pair (E, eq) for $X, Y, f, g\colon X \to Y$ is...

- ...an equalizer if the universal property given by the left diagram below holds
- ...a coequalizer if the universal property given by the right diagram below holds



- These might not exists
- If they exist, then they are unique up to unique isomorphism
- The notions equalizer and coequalizer are dual

More equalizers



- Top The equalizer of f and 0 is the kernel
- Bottom The equalizer of f and "map everything to y" is the preimage of y

Thank you for your attention!

I hope that was of some help.