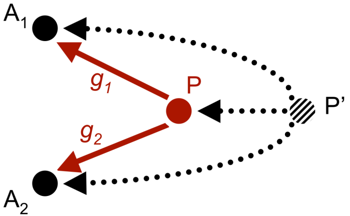
What are...pushouts and pullbacks?

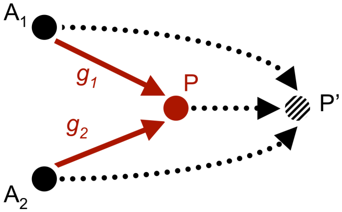
Or: Faces "equal" relations

No faces so far

product:

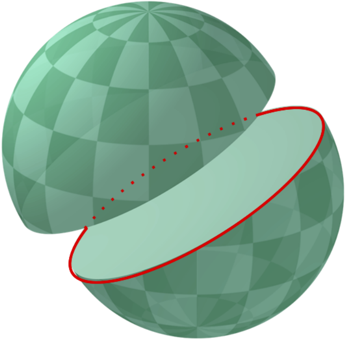


coproduct:



- Products/coproducts have no faces (ignoring the dotted part)
- Products/coproducts are rather naive constructions
- Question What happens if one sees a face?

Faces in topology

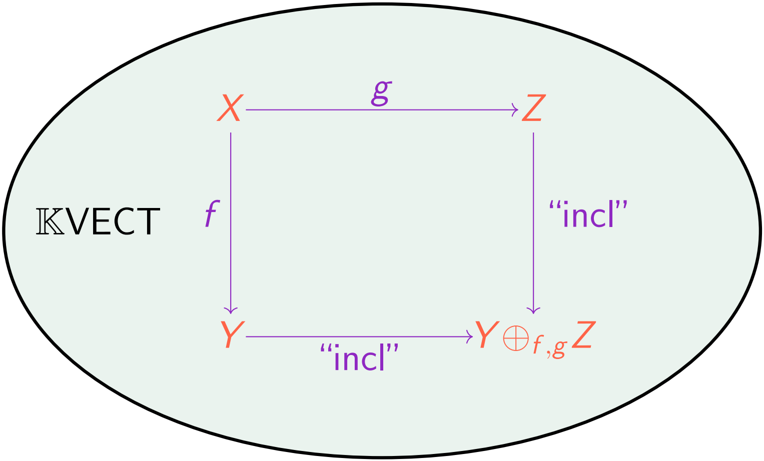


- Hemisphere The pullback of $S^{n-1}, 2 \times D^n$ gives S^n

$$\begin{array}{ccc} S^{n-1} & \xrightarrow{\text{inclusion}} & D^n \\ {\scriptstyle \text{inclusion}}\downarrow & & \downarrow{\scriptstyle \text{gluing}} \\ D^n & \xrightarrow[\text{gluing}]{} & S^n \end{array}$$

- Gluing This works in general

Vector spaces again

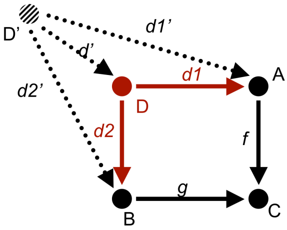


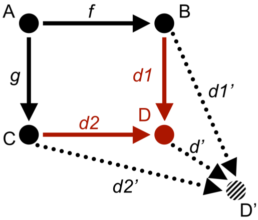
- The above diagram is a “direct sum with gluing”
- More formally, $Y \oplus_{f,g} Z = (Y \oplus Z)/(f(x), -g(x))$
- For $X = 0$ this is just the direct sum

For completeness: A formal definition

An object X with two arrows d_1, d_2 is...

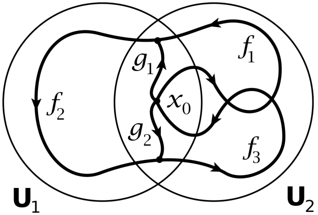
- ...a pullback if the universal property given by the left diagram below holds
- ...a pushout if the universal property given by the right diagram below holds

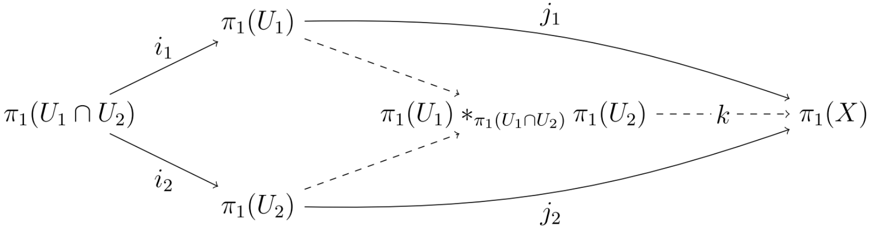




- These might not exists
- If they exist, then they are unique up to unique isomorphism
- The notions pullback and pushout are dual

Seifert-van Kampen is a pushout!





- The Seifert–van Kampen theorem can be stated as a pushout
- The point: the pushout in GROUP is the free product with amalgamation

Thank you for your attention!

I hope that was of some help.