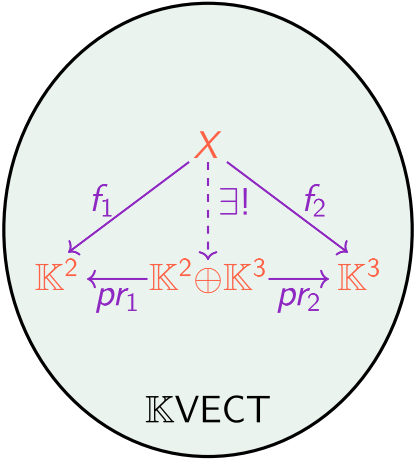
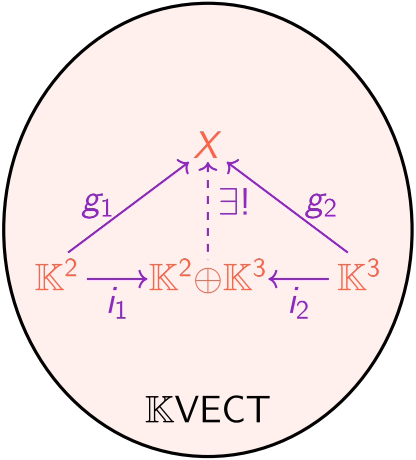
What are...products and coproducts?

Or: Vector spaces rock!

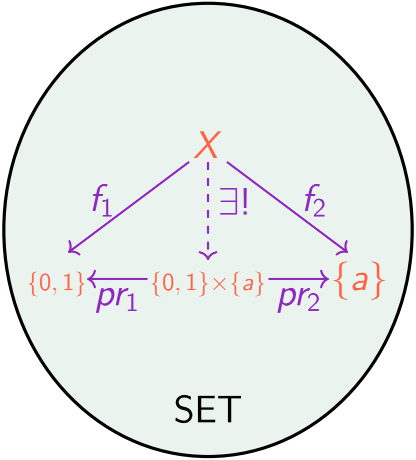
Vector spaces are nice

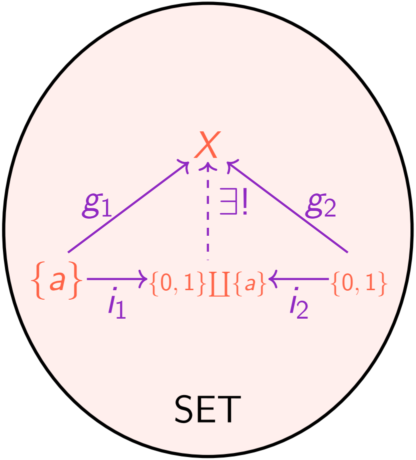




- Every map into $X \oplus Y$ is uniquely determined by what it does on X, Y
- Every map from $X \oplus Y$ is uniquely determined by what it does on X, Y
- No other $\mathbb{K}$-vector space has these properties

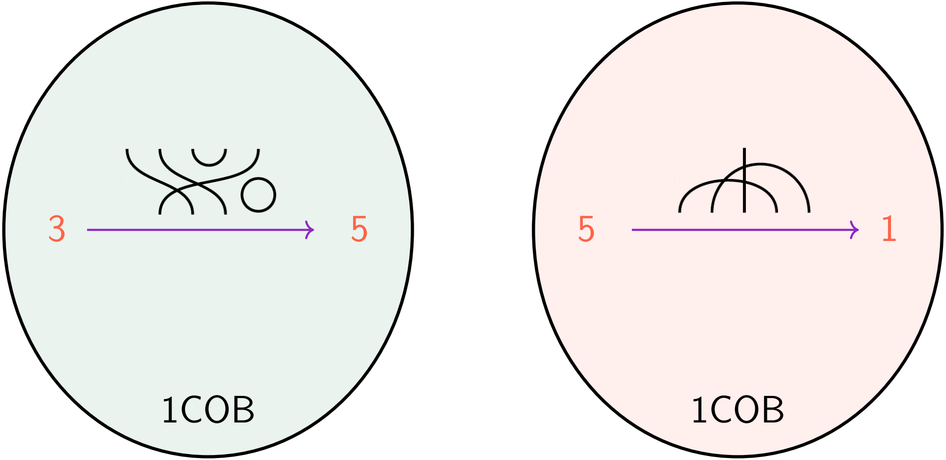
Sets are not quite as nice





- Every map into $X \times Y$ is uniquely determined by what it does on X, Y
- Every map from $X \coprod Y$ is uniquely determined by what it does on X, Y
- No other sets have these properties

Cobordism lack structure

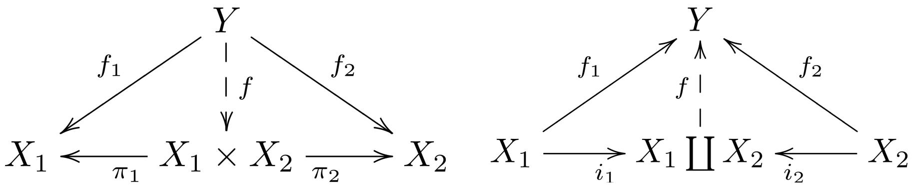


- No object can split ingoing cobordisms into left and right
- No object can split outgoing cobordisms into left and right
- No object of 1COB qualifies as a (co)product

For completeness: A formal definition

An object $X_1 \times X_2 / X_1 \coprod X_2$ together with arrows $\pi_1, \pi_2 / i_1, i_2$ is...

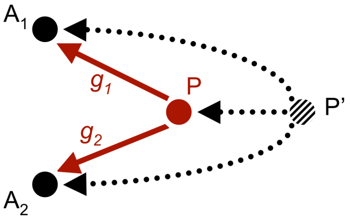
- ...a product if the universal property given by the left diagram below holds
- ...a coproduct if the universal property given by the right diagram below holds
- ...a direct sum if its a product and a coproduct



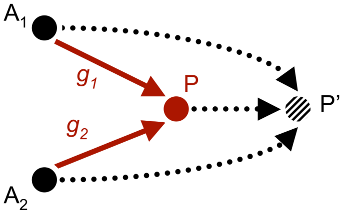
- These might not exists
- If they exist, then they are unique up to unique isomorphism
- The notions product and coproduct are dual, so direct sum is self-dual

Beware infinities!

product:



coproduct:



These can be defined for arbitrary many "factors", but:

- For $\mathbb{K}$VECT these do not agree in general
- But no worries: for finitely many factors they still agree

Thank you for your attention!

I hope that was of some help.