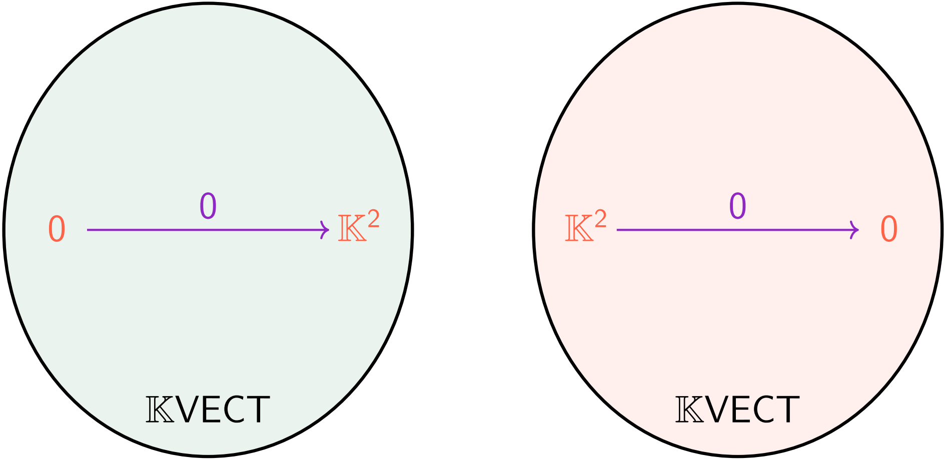
What are...initial and terminal objects?

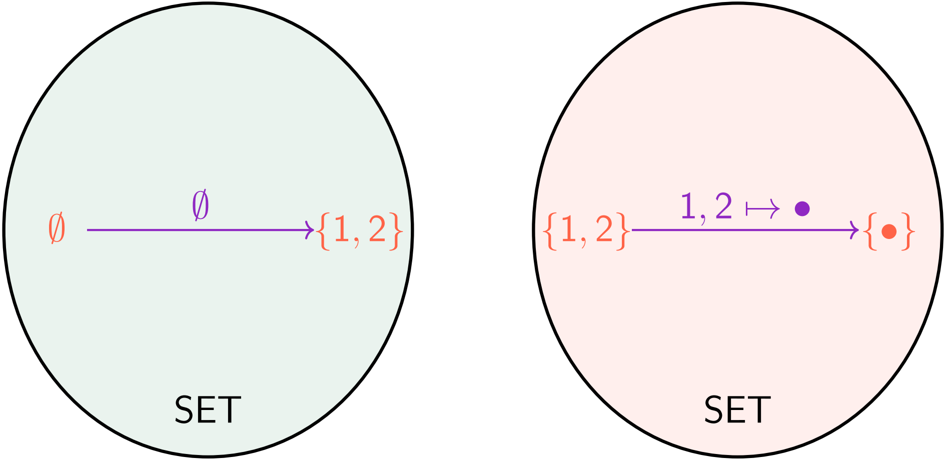
Or: Another reason why zero is great!

Zero!



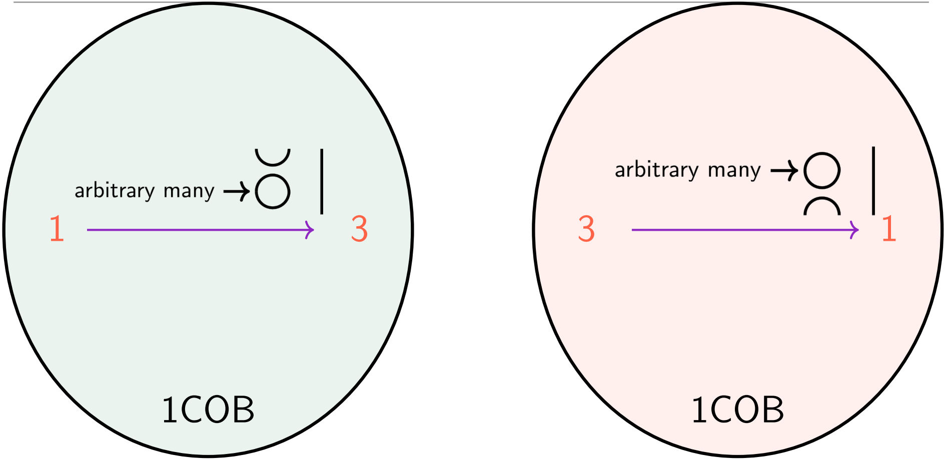
- There is precisely one map $0 \to X$ for any $\mathbb{K}$-vector space X
- There is precisely one map $X \to 0$ for any $\mathbb{K}$-vector space X
- No other $\mathbb{K}$-vector space has these properties

Not quite zeros



- There is precisely one map $\emptyset \to X$ for any set X
- There is precisely one map $X \to \{\bullet\}$ for any set X
- No other sets have these properties

No zeros

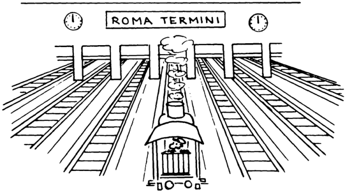


- For any Y there are infinitely many cobordisms $Y \to X$ for any X in 1COB
- For any Y there are infinitely many cobordisms $X \to Y$ for any X in 1COB
- No object of 1COB qualifies as a zero

For completeness: A formal definition

An object Y is...

- ... initial if $\exists!$ arrow $Y \to X$ for all X
- ... terminal if $\exists!$ arrow $X \to Y$ for all X
- ... zero if its initial and terminal



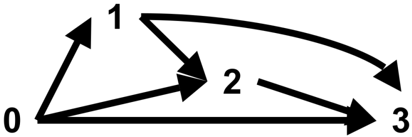
A terminal object

- These might not exists
- If they exist, then they are unique up to unique isomorphism
- The notions initial and terminal are dual, so zero is self-dual

A witness for "easy"

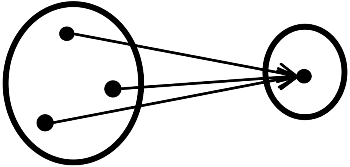
→ The number 0 in this category:

initial:



→ Any singleton set in the category of sets

terminal:



Having initial/terminal objects makes a category "algebraically easy", e.g.:

- $\mathbb{K}$VECT has both Best category ever ;-)
- FIELD has neither Not the nicest category around

Thank you for your attention!

I hope that was of some help.