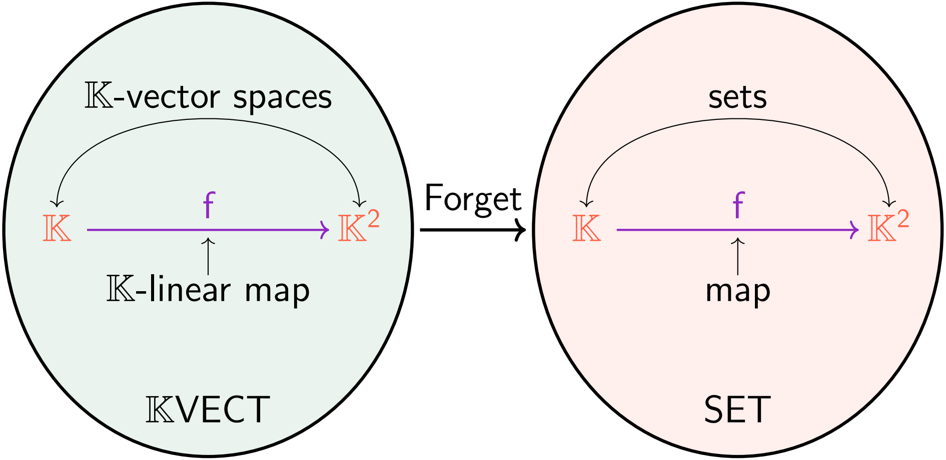
What is...a concrete category?

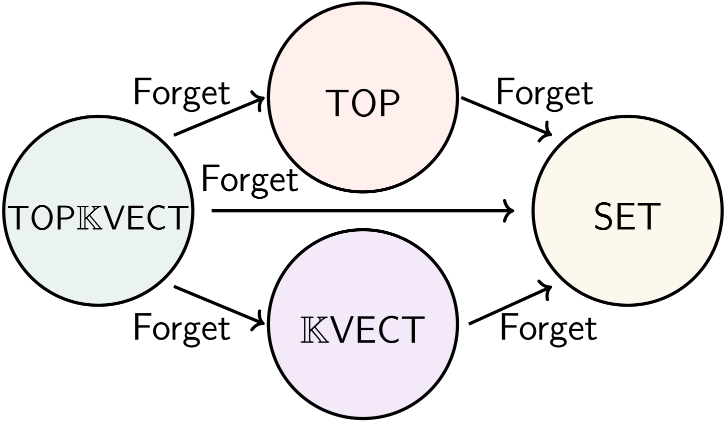
Or: Set-based is cool, well sometimes ;-)

Vector spaces are set-based



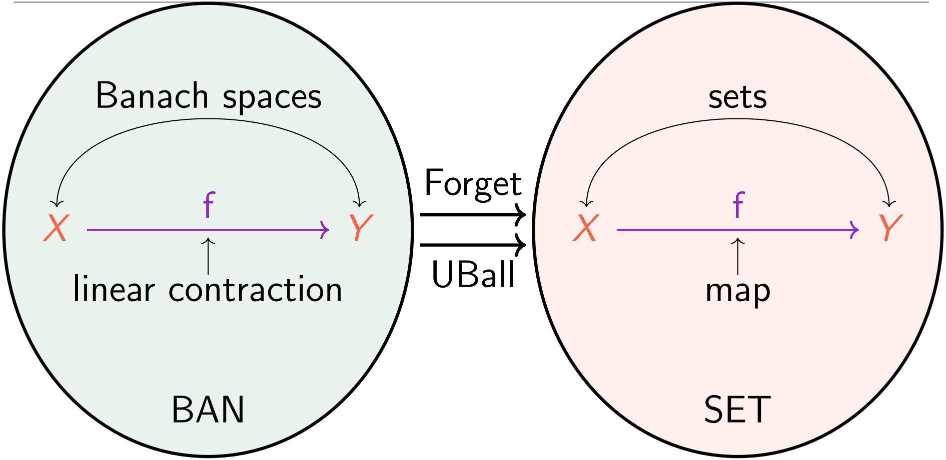
- 𝕂VECT is a set-based category
- Viewing it entirely abstractly ignores some structure
- Idea Maybe view it abstractly together with its underlying sets?

More structure



- TOP$\mathbb{K}$VECT Topological $\mathbb{K}$-vector spaces
- TOP$\mathbb{K}$VECT is based on TOP, $\mathbb{K}$VECT and SET
- Use this to say more about TOP$\mathbb{K}$VECT

Choices what to forget



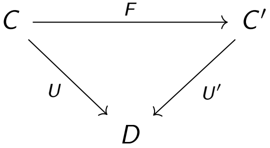
- BAN Banach spaces
- BAN has a forgetful and a unit ball functor to SET
- Both simplify BAN to SET

For completeness: A formal definition

A concrete category over D (the base) is a pair (C, U), where:

- C is a category The richer one
- $U\colon C \to D$ is faithful Like Forget

A concrete functor $F\colon (C, U) \to (C', U')$ (same base D) wants a commutative diagram:



One can now define:

- Equivalence of concrete categories
- The category of concrete categories
- Etc.

Various topologies

The standard descriptions of topological spaces by means of

- neighborhoods,
- open sets,
- closure operators, or
- convergent filters,

give technically different categories, all of which are concretely isomorphic

Thank you for your attention!

I hope that was of some help.