# What is...Whitehead's theorem?

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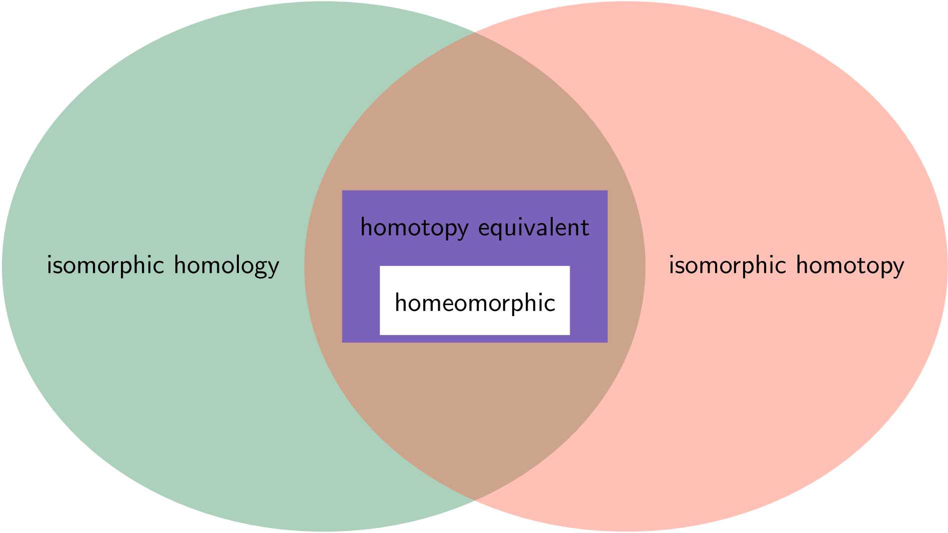
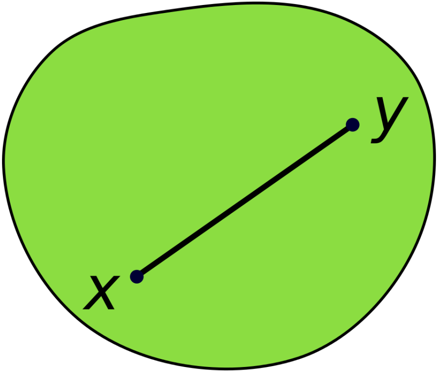
Or: A perfect invariant!?

## How perfect are they?



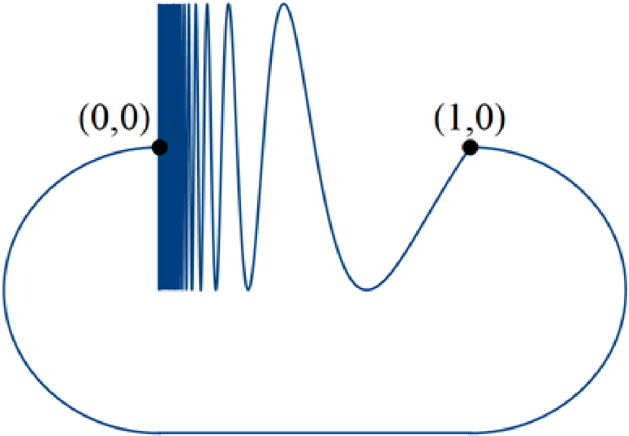

- Homotopy equivalence induce isomorphisms in homotopy/homology
- Question What about the converse?

# The good

$X =$ 


- The space $X$ has trivial homotopy $\pi_*(X) \cong 0$
- The space $X$ is trivial $W \simeq \text{point}$

# The bad

$W =$ 


- The Warsaw circle $W$ has trivial homotopy $\pi_*(W) \cong 0$
- The Warsaw circle $W$ is not trivial $W \not\simeq \text{point}$

## For completeness: A formal statement

For connected cell complexes $X, Y$ and $f\colon X \to Y$ the following are equivalent:

(a) $f\colon X \to Y$ is a homotopy equivalence Topology

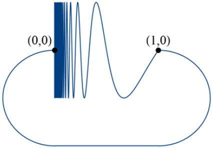
(b) $f_*\colon \pi_*(X) \to \pi_*(Y)$ is an isomorphism Algebra

For connected cell complexes $X, Y$ and $f\colon X \to Y$ the following are equivalent:

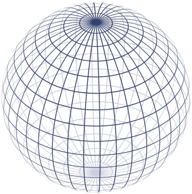
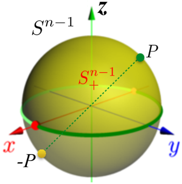
(a) $f\colon X \to Y$ is a homotopy equivalence Topology

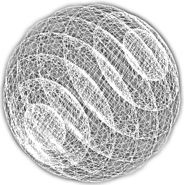
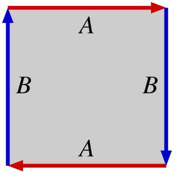
(b) $f_*\colon \pi_1(X) \to \pi_1(Y)$ is an isomorphism and (some) $\tilde{f}\colon \tilde{X} \to \tilde{Y}$ gives an isomorphism $\tilde{f}_*\colon H_*(\tilde{X}) \to H_*(\tilde{Y})$ Algebra

Not a cell complex:

$W =$ 

## The ugly

$X = S^2 \times \mathbb{R}P^3 \cong$  $\times$ 


$Y = S^3 \times \mathbb{R}P^2 \cong$  $\times$ 


- The connected cell complexes $X, Y$ have the same $\pi_*$ $X \simeq Y$ by Whitehead?
- The connected cell complexes $X, Y$ have different $H_*$ $X \not\simeq Y$!
- What fails? There is no $f\colon X \to Y$ inducing all isomorphisms

**Thank you for your attention!**

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I hope that was of some help.