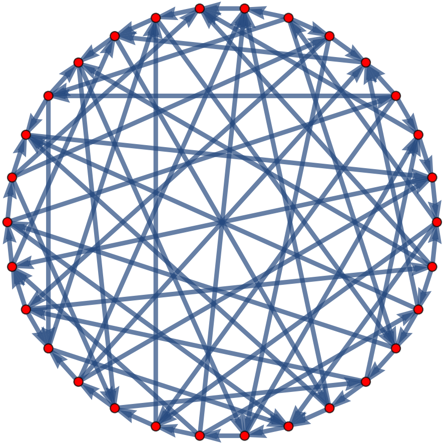
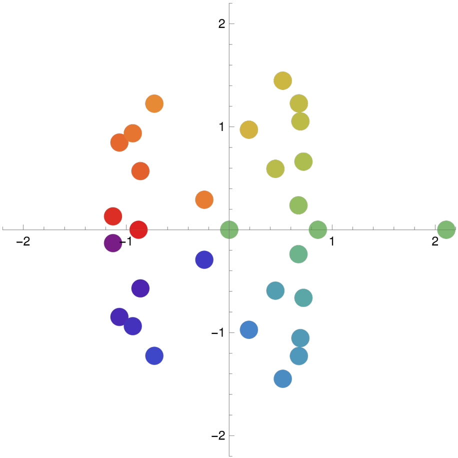
What are...the second eigenvalue's contributions?

Or: Not quite canonical

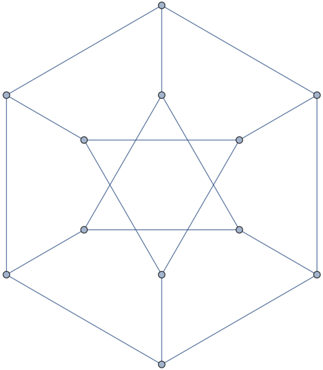
The spectrum - a reminder



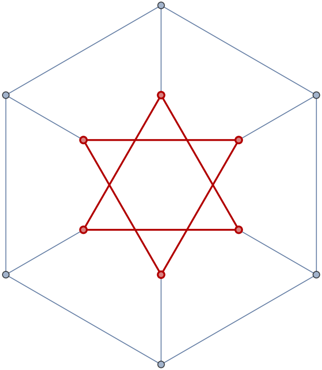


- The leading eigenvalue encodes growth rates of # of paths
- Question What about the other eigenvalues?
- E.g. what about the "second largest" λ_2? (For graphs we have $S(G) \subset \mathbb{R}$)

Connected subgraphs – part one



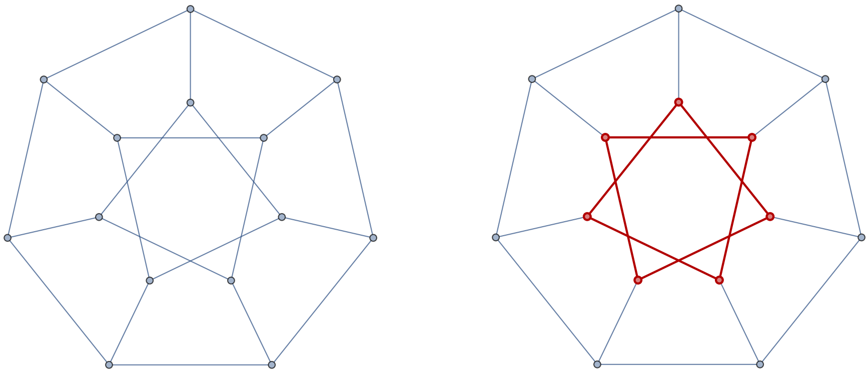
$S \approx \{3, 2.24, 1.41^2, 1, 0^2, -1.41^2, -2^2, -2.24\}$



$S = \{2^2, -1^4\}$

- The difference $\lambda_1 - \lambda_2$ plays an important role
- Above λ_1^H does not fit into $\lambda_2 < x \leq \lambda_1$ for H=subgraph
- H is not connected

Connected subgraphs – part two



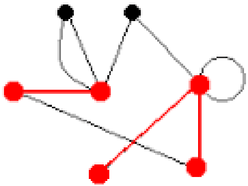
$S \approx \{3, 1.71^2, 1.55^2, 1, 0.08^2, -0.91^2, -2.10^2, -2.33^2\}$ $\quad$ $S \approx \{2, 1.25^2, -0.45^2, -1.80^2\}$

- The difference $\lambda_1 - \lambda_2$ plays an important role
- Above λ_1^H does fit into $\lambda_2 < x \leq \lambda_1$ for H=subgraph
- H is connected

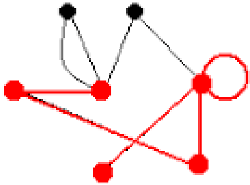
For completeness: A formal statement

Let G be a graph with second-largest eigenvalue λ_2. Let H be a nonempty regular induced subgraph with largest eigenvalue $\lambda_2 < \lambda_1^H$. Then H is connected

- Regular = every vertex has the same degree
- induced subgraph = take all edges for a fixed set of vertices



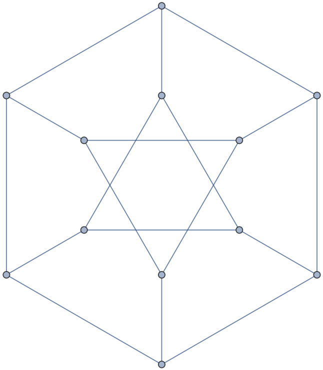
Subgraph (in red)



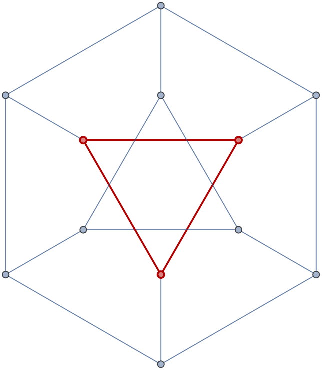
Induced Subgraph

- Since the second largest eigenvalue is "less canonical" than the PF eigenvalue, we should expect weaker statements

Not an "If and only if"



$S \approx \{3, 2.24, 1.41^2, 1, 0^2, -1.41^2, -2^2, -2.24\}$



$S = \{2, -1^2\}$

- The difference $\lambda_1 - \lambda_2$ plays an important role
- Above λ_1^H does not fit into $\lambda_2 < x \leq \lambda_1$ for H=subgraph
- H is connected

Thank you for your attention!

I hope that was of some help.