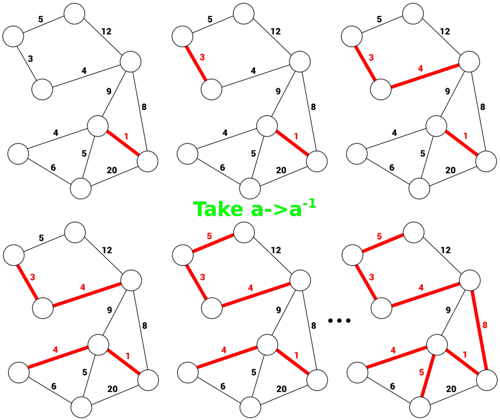
What is...a greedy algorithm 2?

Or: Greedy for matroids

Greedy strategy for spanning forests

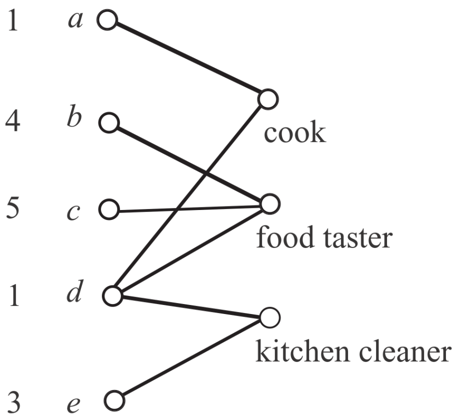


- Maximal spanning forests can be found using a greedy strategy
- Maximal spanning forests are the (weighted) bases of a matroid
- Crucial observation This is not a coincidence

Matroid optimization problem

Task: hire the best candidates (a,b,c,d,e) for the jobs (cook *etc.*):

weighting = ranking



- Task For a matroid with weight function $E \to \mathbb{R}$, find a basis of maximum weight
- Example Maximal spanning forests for the graphic matroids (last slide)
- Examples Maximal bipartite matchings for the transversal matroids (above)

The greedy strategy

Greedy algorithm

Input: A finite set E, a weight function $w : E \to \mathbb{R}$ and a family $\mathcal{I}$ of subsets of E.

Order the elements of E: $e_1, e_2, \ldots, e_n$ so that $w(e_i) \geq w(e_j)$ for $i \leq j$.

Set $B := \emptyset$.

For $i = 1$ to n,

$$\text{if } B \cup e_i \in \mathcal{I}, \text{ then set } B := B \cup e_i.$$

Output: B, a maximal member of $\mathcal{I}$ of maximum weight.

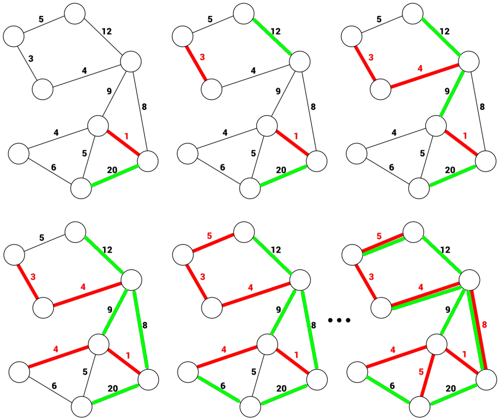
- Matroid $(E, \mathfrak{I} = \mathcal{I})$ via linear independent sets
- Weighting $=$ ranking
- Example On the previous page we get $\{c, e, a\}$ (or $\{c, e, d\}$)

For completeness: A formal statement

The greedy algorithm works for all matroids and all weightings

- One can characterize when a greedy strategy applies (more on the next slide)
- Both, maximal or minimal, can be done similarly

This is often
stated as
a minimal spanning
forests problem
:



Matroid embeddings

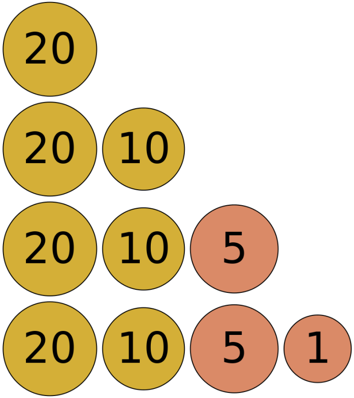
Not related to a matroid:

$36 - 20 = 16$

$16 - 10 = 6$

$6 - 5 = 1$

$1 - 1 = 0$



- Very often greedy situations come from a matroid but not all
- There is a generalization of a matroid called matroid embedding such that all greedy situations come from these matroid embeddings

Thank you for your attention!

I hope that was of some help.