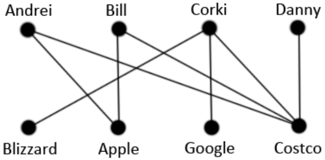
What are...transversal matroids?

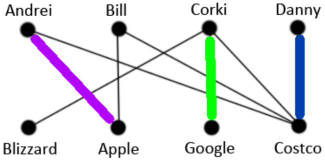
Or: More examples from graph theory

Matchings

A graph:



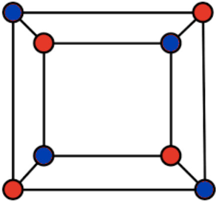
A matching:



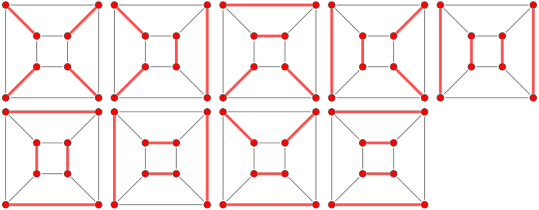
- Recall that a matching is a set of edges without common vertices
- Example Matching employees with companies
- Simplification No employ can have two companies; none of our employs work for the same company

Maximal matchings

A graph:



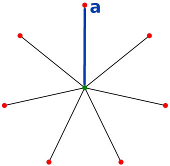
All maximal matchings:



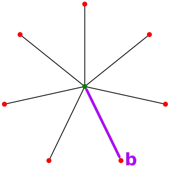
- Observation For a bipartite graph all maximal matchings have the same size
- Maximal = cannot add more edges
- This reminds us of bases, right?

Exchange of vertices

A matching:



Another matching:



- If we consider monochromatic vertices in a maximal matching then the basis exchange property (BEP) holds
- Recall the BEP For $A \neq B$ in $\mathfrak{B}$ and $a \in A \setminus B$ there exists $b \in B$ such that $(A \setminus \{a\}) \cup \{b\} \in \mathfrak{B}$
- This now really reminds us of bases, right?

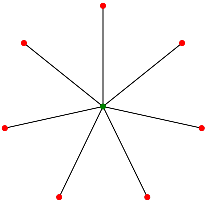
For completeness: A formal statement

The transversal matroid associated to a bipartite graph $G = (V = X \coprod Y, E)$ has:

(i) Linear independent sets are the vertices in X that are part of a matching

(ii) Bases are the same but for maximal matching

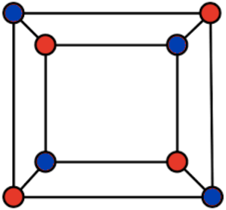
Example The star graph



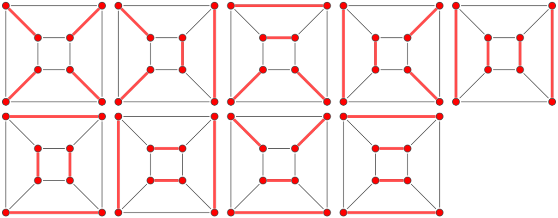
- In this case the nonsilly linear independent sets are the outside vertices as a single set
- All of these are also bases

The other extreme

A graph:



All maximal matchings:



- Star All linear independent sets are bases
- Cube There is only one basis (arising from many matchings!)

Thank you for your attention!

I hope that was of some help.