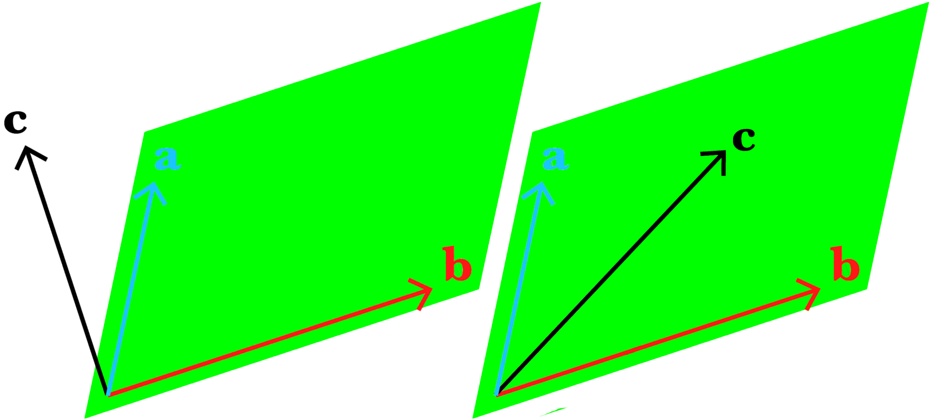
What are...linearly independent sets?

Or: All bases are of the same size

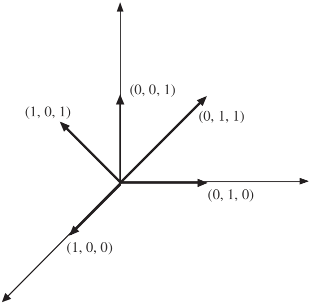
Linear independence (LI)



- Matroids generalize bases of vector spaces
- Bases are maximal with respect to linear independence
- Task Define linear independence for matroids

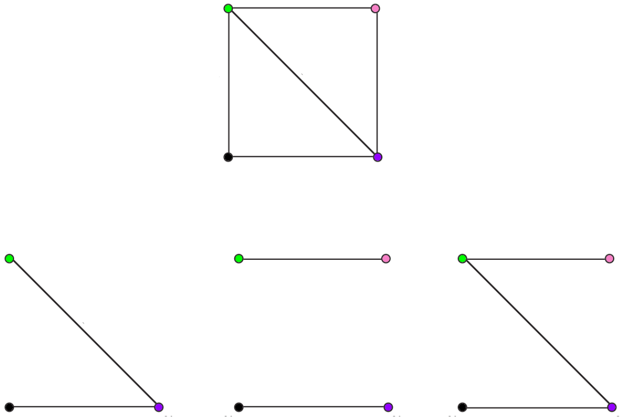
Linear matroids and LI

$$\begin{array}{ccccc} a & b & c & d & e \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$



- In a linear matroid LI is what you think it is
- Example We have for example $\{a, c, e\}$ and $\{c, e\}$ are linearly independent
- The maximal ones are bases, for example $\{a, c, e\}$ is a basis

Graphic matroids and LI



- In a graphic matroid LI is a set of edges "having no cycle"
- Example We have, for example, the linear independent sets above
- The maximal ones are spanning forest (= touching all vertices), for example the right one is a spanning forest

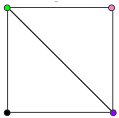
For completeness: A formal statement

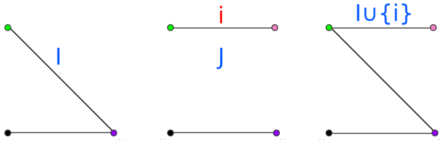
A **matroid** is a pair $(E, \mathfrak{I})$ of a finite set E and LI sets $\mathfrak{I} \subset \mathfrak{P}(E)$ such that:

(i) $\mathfrak{I}$ is not empty **Existence of LI sets**

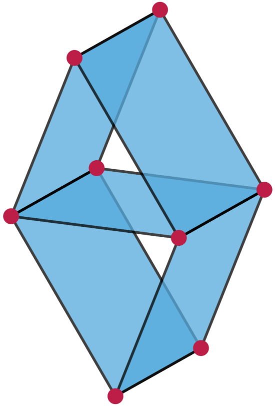
(ii) For $I \subset J$ and $J \in \mathfrak{I}$ implies $I \in \mathfrak{I}$, and for $|I| < |J|$ there exists $i \in J \setminus I$ such that $I \cup \{i\} \in \mathfrak{I}$ **Vector exchange property**

- This definition is **equivalent** to the basis version; basis = max LI set (always of the same size)
- Here is the vector exchange property for a graphic matroid:





A nonlinear and nongraphic example



- Vamos matroid $E =$ the points, $\mathfrak{I} =$ all subsets of points of order < 4 and the ones not marked in the picture above
- Bases are the not marked collection of four points
- This example is neither linear nor graphic

Thank you for your attention!

I hope that was of some help.