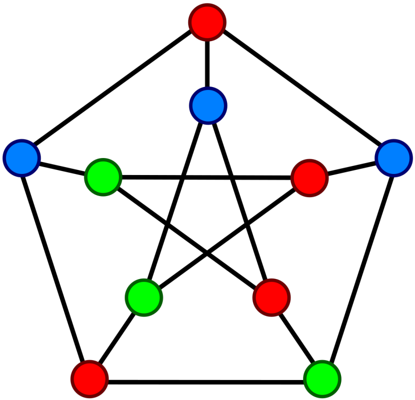
What are...colorings of random graphs?

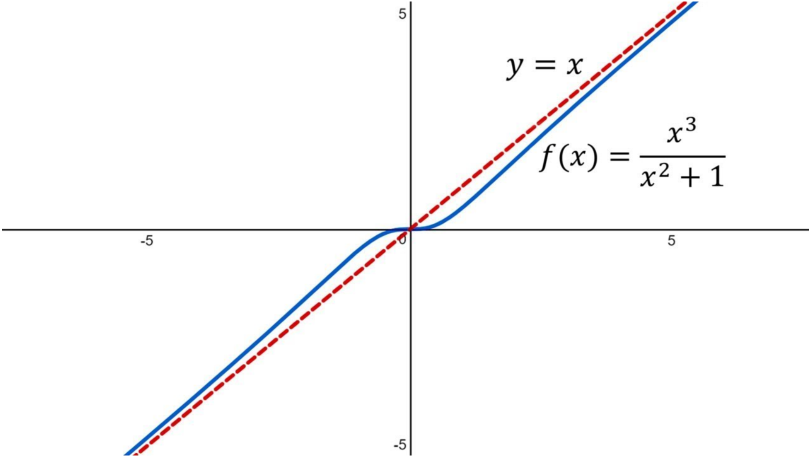
Or: Concentrated colors

Colorings



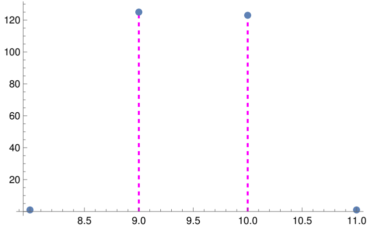
- Coloring = coloring vertices such that adjacent vertices get different colors
- Chromatic number $\chi(G)$ = minimal number of colors needed for colorings
- Question $\chi(G)$ is terribly difficult to compute, so is there any chance to do this for random graphs, say for $G_{n,1/2}$?

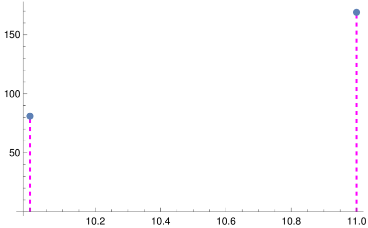
This is very difficult!



- Question Can we get a nice formula for the asymptotics of $\chi(G_{n,1/2})$?
- This seemingly innocent question was open for decades in spite of serious efforts

Very concentrated





- Above χ of 250 $G_{50,1/2}$ and $G_{60,1/2}$
- There seems to be a concentration around one or two values $\approx n/2 \log_2 n$

For completeness: A formal statement

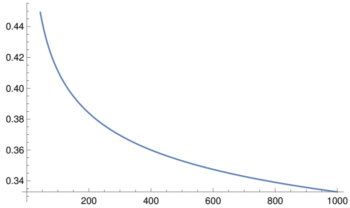
For $d = 1/(1-p)$ we have almost all $G_{n,p}$ concentrated in the interval

$$\chi(G_{n,p}) \in \frac{n}{2\log_d n}\left[1+f, 1+3f\right]$$

where we have the following threshold function

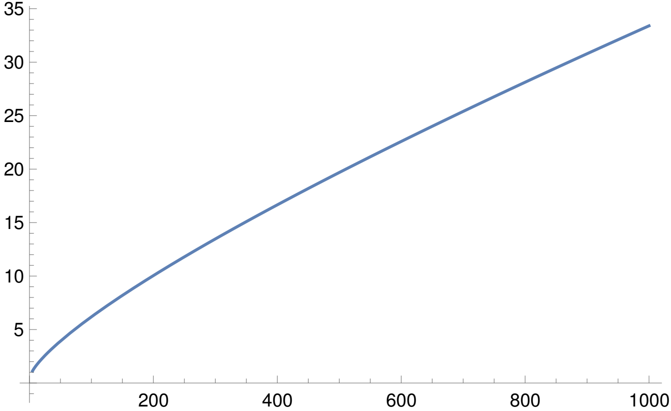
$$f = \log_2 \log_2 n / \log_2 n$$

- This is saying that $\chi(G_{n,p}) \approx n/2\log_d n$
- Also: $\chi(G_{n,p})$ is concentrated in some interval, depending on n, p
- Here is a plot for $p = 1/2$ of the threshold function



Compare with $c(G_{n,p})$

Int. length:



- The interval on the previous page is quite large, see above
- For the clique number $c(G_{n,p})$ we have seen the same phenomena and got a much better statement
- For χ one expects a better statement but this is/seems open

Thank you for your attention!

I hope that was of some help.