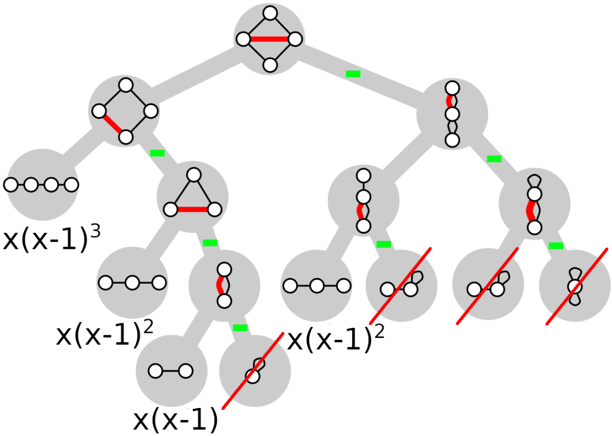
What is...the Tutte polynomial?

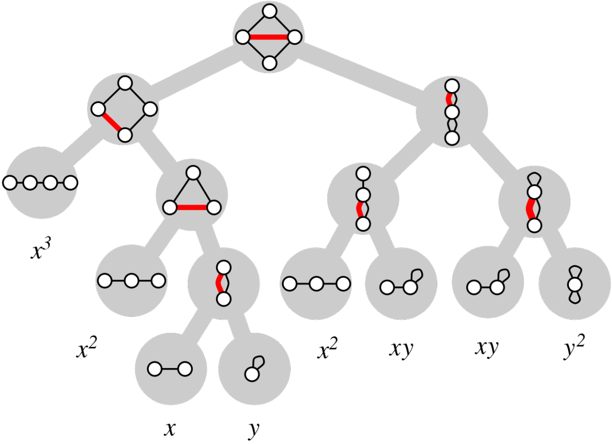
Or: More counting!

The chromatic polynomial



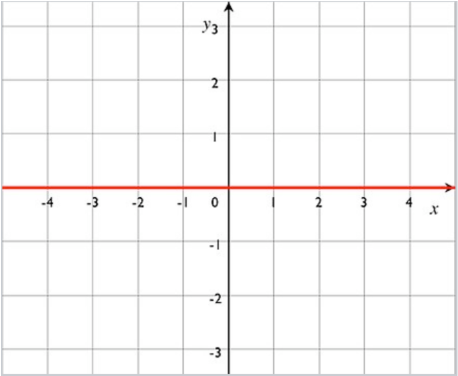
- Recall that a way to define the chromatic polynomial was deletion-contraction
- Here we kill loops since loops rule out colorings
- Idea Keep the loops, give them the variable y so that $P_G(x,0) = P_G(x)$

The Tutte polynomial



- Here is an algorithm to compute $P_G(x, y)$ = Tutte polynomial
- Starting condition $P_{tree}(x, y) = x^{\#vertices-1}$ and $P_{loop}(x, y) = y$
- Then use deletion-contraction: $P_G(x, y) = P_{G \setminus e}(x, y) + P_{G/e}(x, y)$

Ok, this one is a bit annoying...



The chromatic polynomial drawn in the Tutte plane

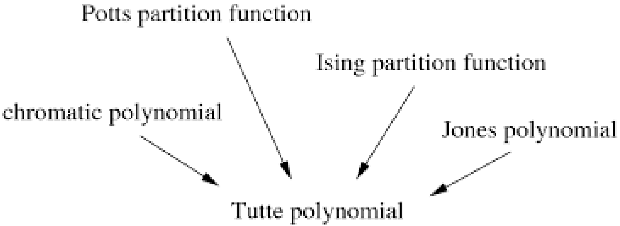
- What one should keep in mind is chromatic(x) = Tutte(x,0)
- However, that is not quite correct
- Correct: chromatic(x) $= (-1)^s x^t$Tutte(x-1,0) with explicit s, t

For completeness: A formal statement

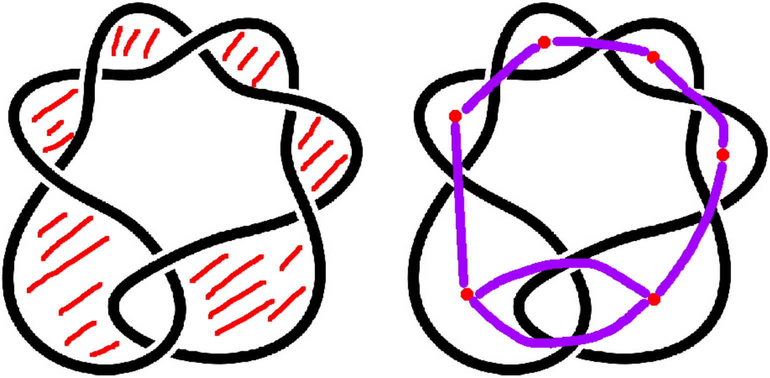
There exists a polynomial $T_G(x, y)$ associated to a graph such that:

- $T_G(2, 1) = \#$ forests
- $T_G(1, 1) = \#$ spanning forests
- $T_G(1, 2) = \#$ spanning subgraphs
- More...

- The polynomial is called Tutte polynomial
- Also we have the specialization "chromatic(x) = Tutte(x,0)", and more



Tutte knows knots



- Step 1 Checkerboard color and alternating knot K
- Step 2 Create the dual graph $G(K)$
- Step 3 $P_{G(K)}(-x, -1/x)$ is the Jones polynomial of K up to scaling

Thank you for your attention!

I hope that was of some help.