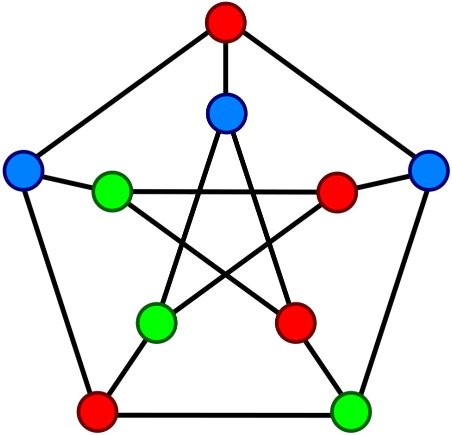
What is...chromatic detection?

Or: Determined by colorings

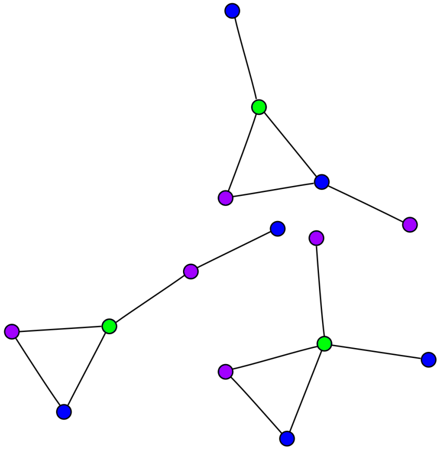
Counting colorings – reminder

3-coloring:



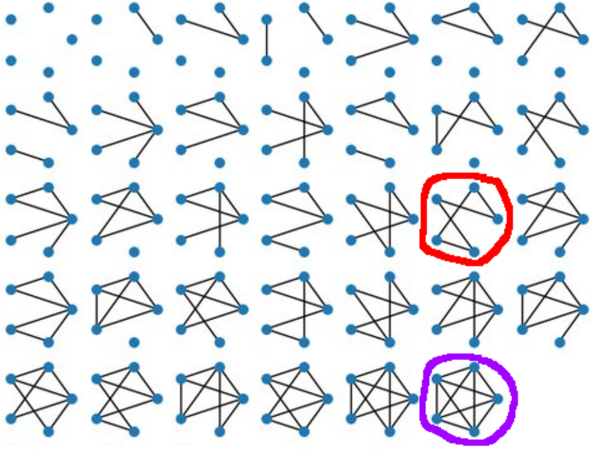
- Recall that the chromatic polynomial $P_G(x)$ counts graph colorings
- It is a graph invariant *i.e.* $(G \cong H) \Rightarrow (P_G(x) = P_H(x))$
- Question What about the converse?

The converse is false



- All the above graphs have twenty-four 3-colorings
- All of them have $P_G(x) = (x-2)(x-1)^3 x$
- They are nonisomorphic

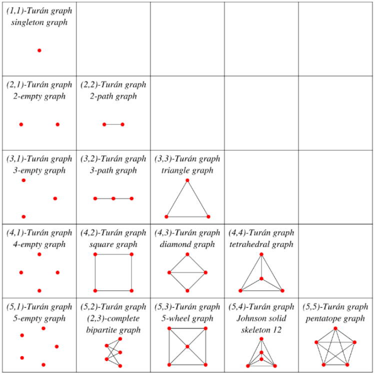
Well, something is true



- There are 34 graphs with 5 vertices see above
- Only the cycle has chromatic polynomial $x(x-1)(x-2)(x^2-2x+2)$
- Only the complete graph has chromatic polynomial $x(x-1)(x-2)(x-3)(x-4)$

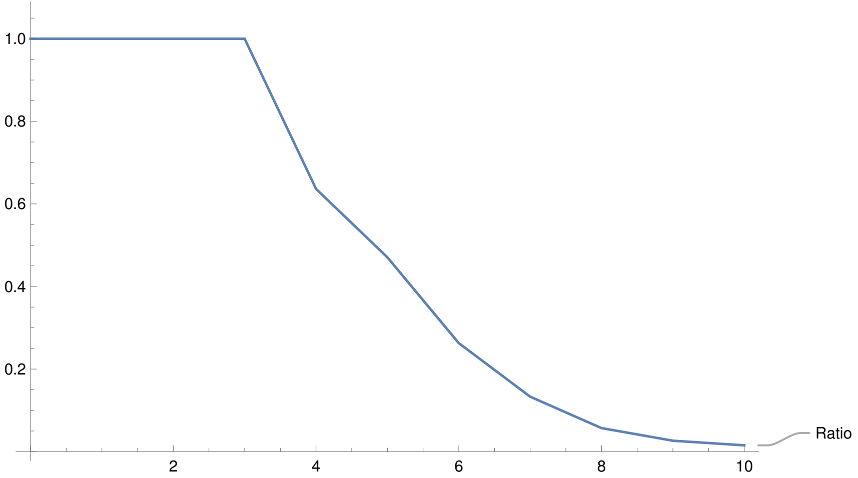
For completeness: A formal statement

Cycles and Turán graphs are chromatically unique



- Chromatically unique (cu) is $(G \cong H) \Leftarrow (P_G(x) = P_H(x))$
- $\#V_G \neq \#V_H$ implies $P_G(x) \neq P_H(x)$, so that part is boring
- Turán graphs include: complete (plain, bipartite or tripartite) and empty graphs

Most graphs are not cu



- There are many more families of cu graphs, but the overall number is (probably) small
- Above the ratio cu graphs/all graphs on n vertices
- I am not aware of any formal statement

Thank you for your attention!

I hope that was of some help.