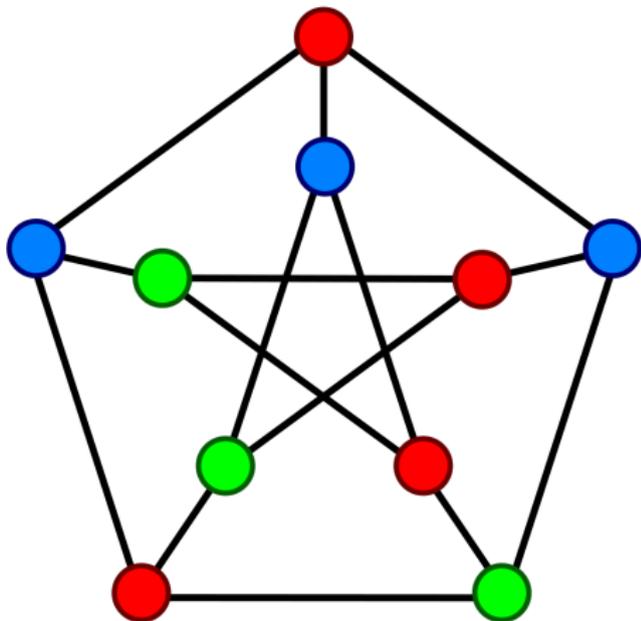


What is...chromatic detection?

Or: Determined by colorings

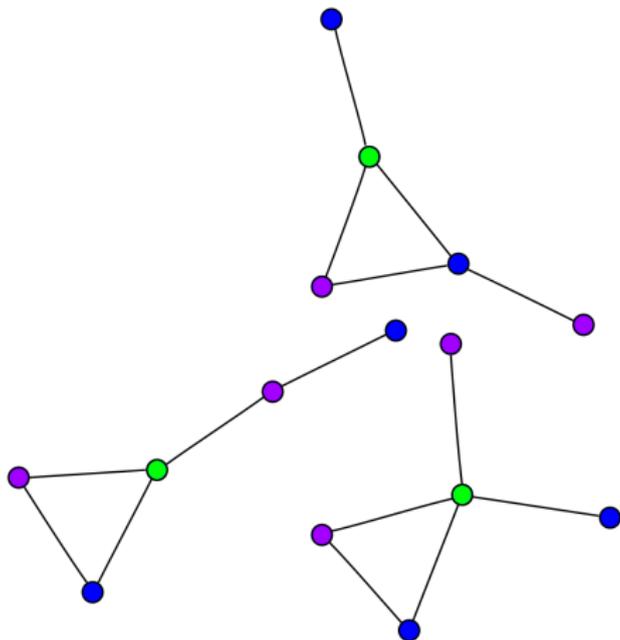
Counting colorings – reminder

3-coloring:



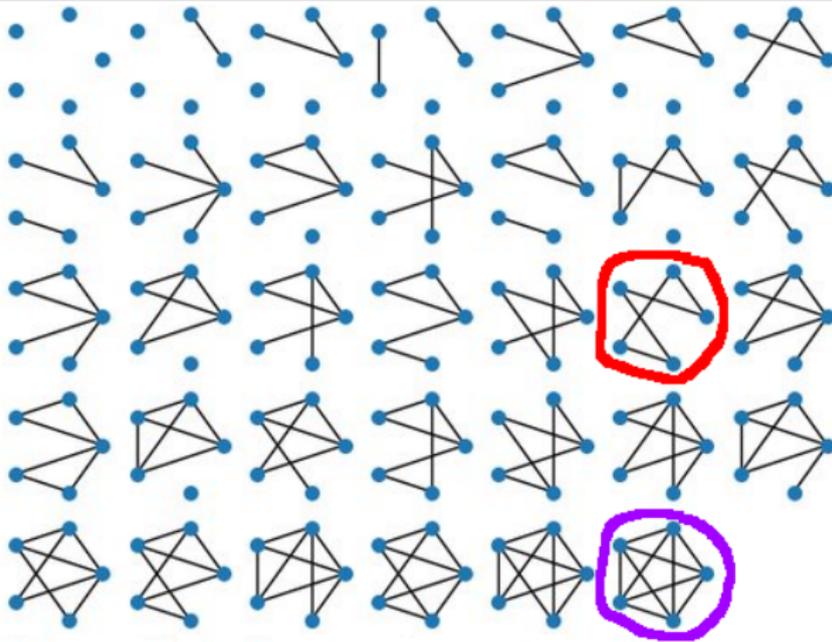
-
- ▶ Recall that the chromatic polynomial $P_G(x)$ counts graph colorings
 - ▶ It is a graph invariant i.e. $(G \cong H) \Rightarrow (P_G(x) = P_H(x))$
 - ▶ Question What about the converse?

The converse is false



-
- ▶ All the above graphs have twenty-four 3-colorings
 - ▶ All of them have $P_G(x) = (x - 2)(x - 1)^3x$
 - ▶ They are nonisomorphic

Well, something is true



- ▶ There are 34 graphs with 5 vertices see above
- ▶ Only the cycle has chromatic polynomial $x(x-1)(x-2)(x^2-2x+2)$
- ▶ Only the complete graph has chromatic polynomial $x(x-1)(x-2)(x-3)(x-4)$

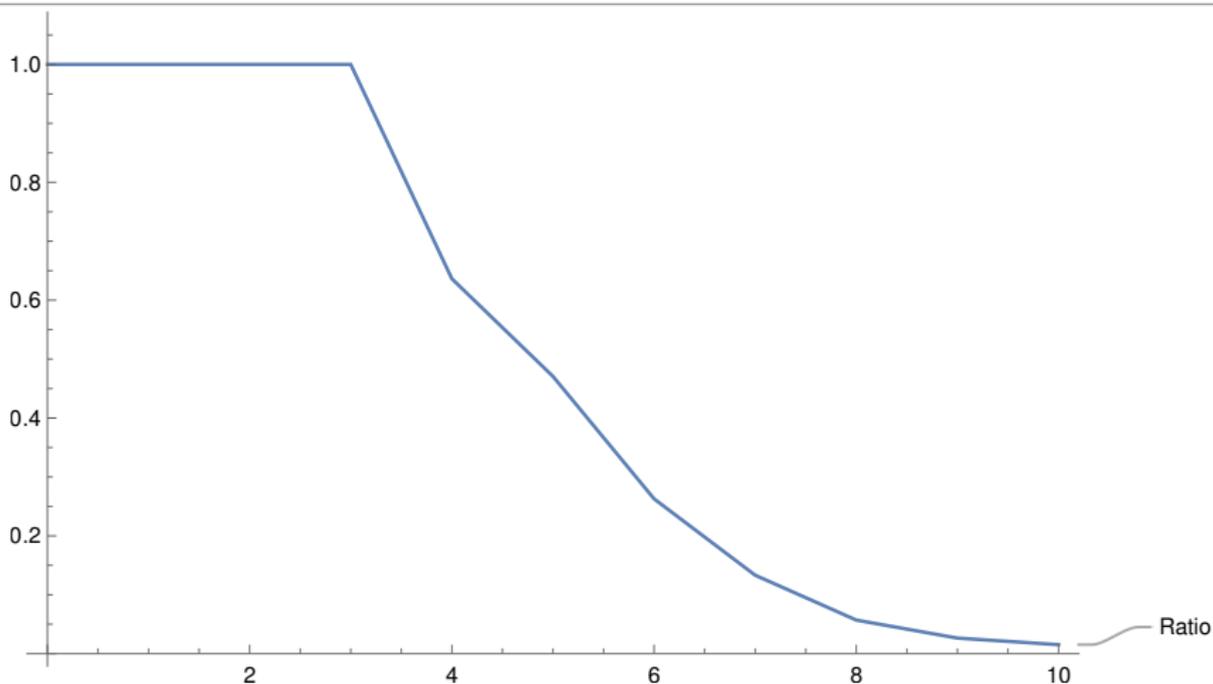
For completeness: A formal statement

Cycles and Turán graphs are chromatically unique

(1,1)-Turán graph singleton graph 				
(2,1)-Turán graph 2-empty graph 	(2,2)-Turán graph 2-path graph 			
(3,1)-Turán graph 3-empty graph 	(3,2)-Turán graph 3-path graph 	(3,3)-Turán graph triangle graph 		
(4,1)-Turán graph 4-empty graph 	(4,2)-Turán graph square graph 	(4,3)-Turán graph diamond graph 	(4,4)-Turán graph tetrahedral graph 	
(5,1)-Turán graph 5-empty graph 	(5,2)-Turán graph (2,3)-complete bipartite graph 	(5,3)-Turán graph 5-wheel graph 	(5,4)-Turán graph Johnson solid skeleton 12 	(5,5)-Turán graph pentatope graph 

- ▶ Chromatically unique (cu) is $(G \cong H) \Leftrightarrow (P_G(x) = P_H(x))$
- ▶ $\#V_G \neq \#V_H$ implies $P_G(x) \neq P_H(x)$, so that part is boring
- ▶ Turán graphs include: complete (plain, bipartite or tripartite) and empty graphs

Most graphs are not cu



- ▶ There are many more families of cu graphs, but the overall number is (probably) small
- ▶ Above the ratio **cu graphs/all graphs** on n vertices
- ▶ I am not aware of any formal statement

Thank you for your attention!

I hope that was of some help.