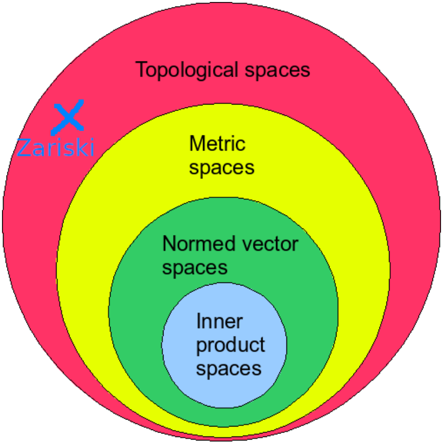
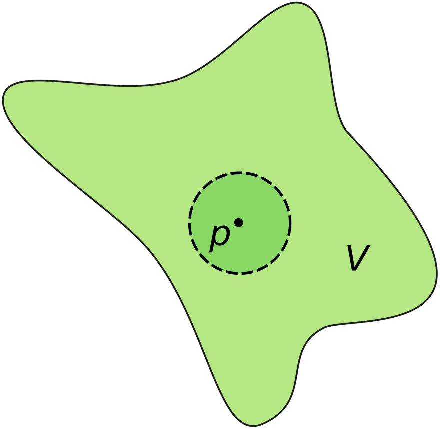
What is...the Zariski topology?

Or: Varieties are closed, by definition

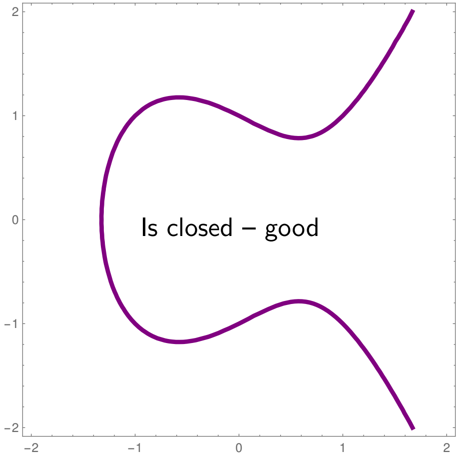
Topological space

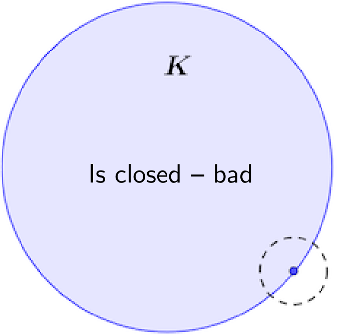




- Topological space = a set with specified so-called open/closed sets
- The crux Such spaces allow the notion of "closeness" or "neighbors"
- Today Zariski's topology which is quite an obscure topology

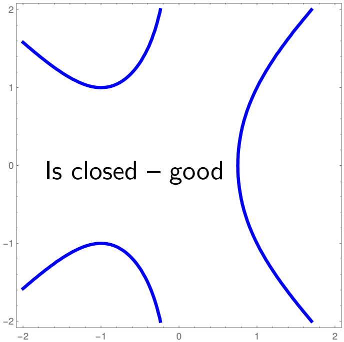
Why not use the Euclidean topology?

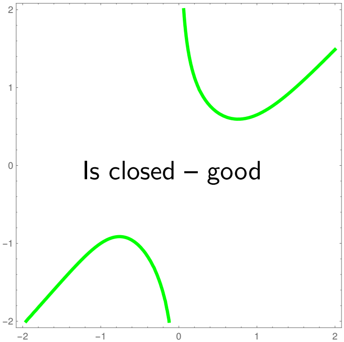




- Observation In the standard topology on $\mathbb{R}^n$ varieties are closed
- Problem Way too many other non-variety-thingies are also closed

Define it to work





- Define closed sets to be varieties
- Why is this a topology? Because:
  - P gets smaller $\leftrightsquigarrow$ V gets bigger:

$$P \subset Q \Rightarrow V(P) \supset V(Q)$$
$$V(P \cup Q) = V(P) \cap V(Q)$$
$$V(PQ) = V(P) \cup V(Q)$$

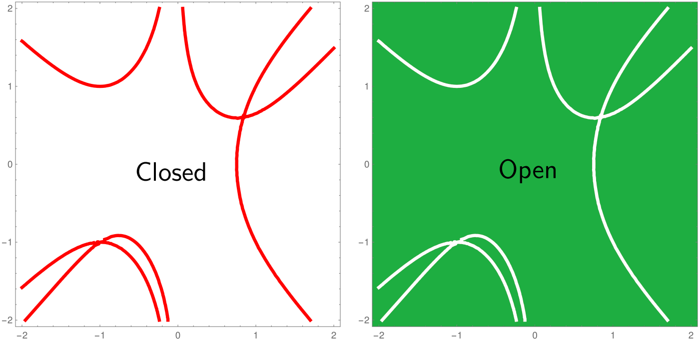
For completeness: A formal statement

The Zariski topology on some affine variety X has exactly the affine subvarieties of X as closed sets

This indeed defines a topology

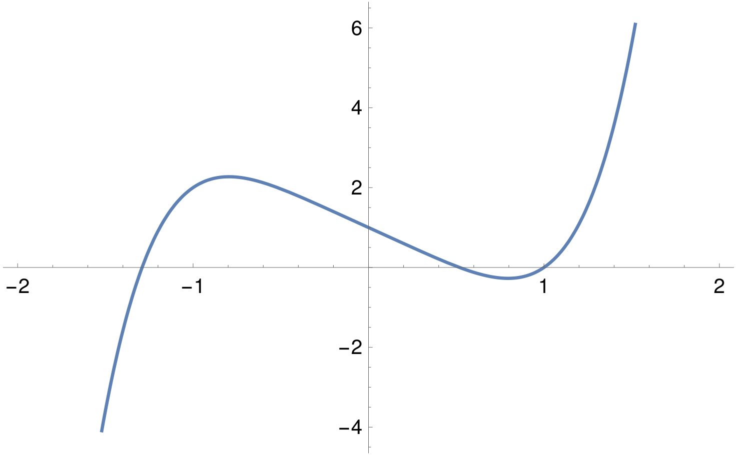
The Zariski topology is a bit obscure, e.g.:

- Closed sets are very small, open sets are very large



- One cannot separate points

The Zariski topology on $\mathbb{R}$



- Subvariety of $\mathbb{R}$ = a finite collection of points
- Why? Because a polynomial in one variable has only finitely many roots
- Closed sets = finite sets and $\mathbb{R}$ itself

Thank you for your attention!

I hope that was of some help.