What is a...Cayley graph?

Or: Graphs and groups

Groups encoded efficiently

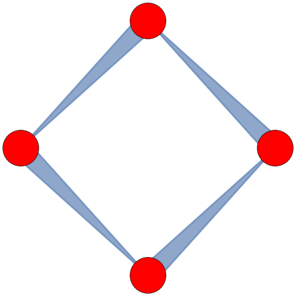
$\mathbb{Z}/4\mathbb{Z}$ (written additively):

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

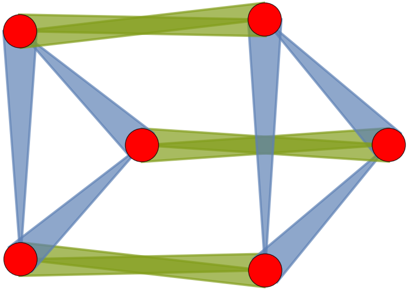
1 is a generator of $\mathbb{Z}/4\mathbb{Z}$:

$$\emptyset = 0, \quad 1 = 1, \quad 11 = 1 + 1 = 2, \quad 111 = 1 + 1 + 1 = 3$$

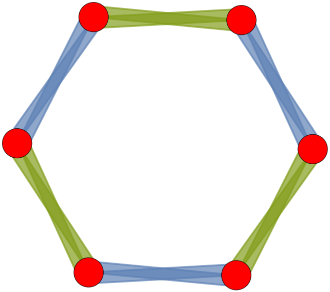
Illustrated as a graph:



Can we recognize the group?



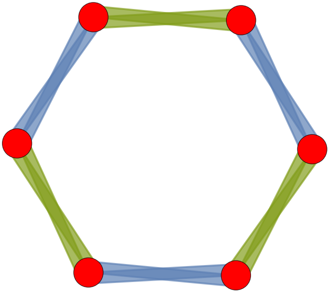
$\leftrightsquigarrow \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$



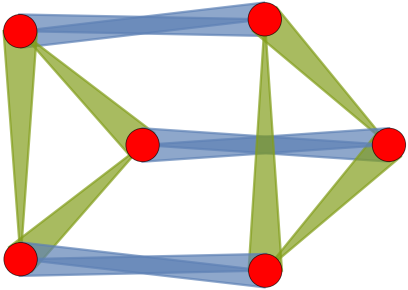
$\leftrightsquigarrow S_3$

Note that we need to illustrate different generators by different colors!

Catch. The graph depends on the chosen generators



$\leftrightsquigarrow S_3 = \langle (1,2), (2,3) \rangle$



$\leftrightsquigarrow S_3 = \langle (1,2), (1,3,2) \rangle$

For completeness: A formal definition

For a group $G = \langle S \rangle$ the Cayley graph $\Gamma = \Gamma(G, S)$ is constructed by:

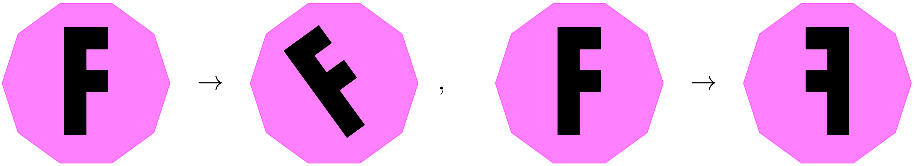
(a) The vertex set of Γ is G

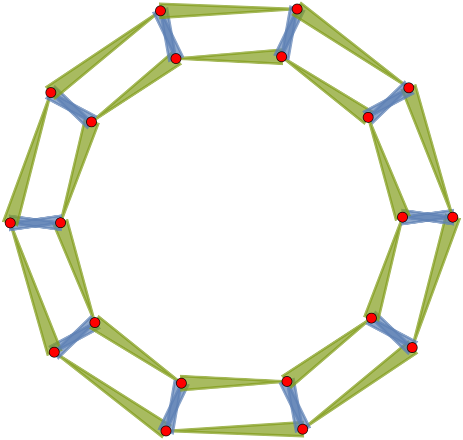
(b) Each $s \in S$ is assigned a color s

(c) Draw an edge of color s from g to gs

- Generators with $s = s^{-1}$ correspond to double edges
- Cayley graphs are strongly connected
- G is commutative if and only if two-step-walks commute Commutative
- Closed walks are relations among words Relations
- A group can thus be studied via its adjacency matrix Linear algebra

Cayley graphs of Sym(polygon) are polygons





Thank you for your attention!

I hope that was of some help.