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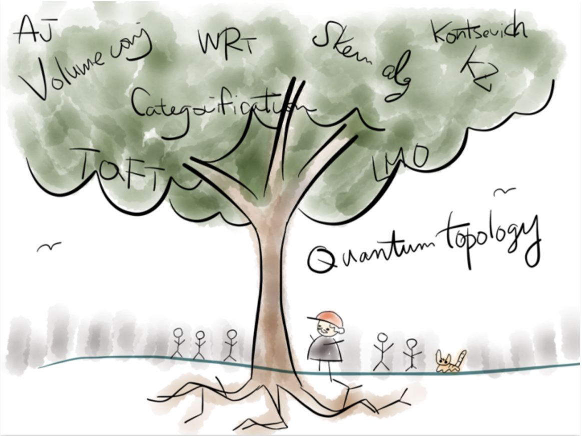

Figure : The quantum algebra tree.

(There is a whole comic of about 10 slides: `http://www.kurims.kyoto-u.ac.jp/~sakie/sakieKyoto/Talks_files/LNsakie.pdf`.)

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Figure : The Jones revolution.

(There is a whole comic of about 10 slides: http://www.kurims.kyoto-u.ac.jp/~sakie/sakieKyoto/Talks_files/LNsakie.pdf.)

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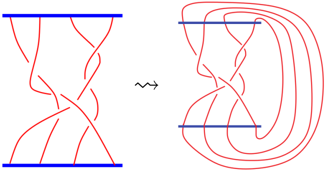

Figure : From braids to knots/links.

(Pictures from "Vaughan Jones, On the origin and development of subfactors and quantum topology".)

The relations

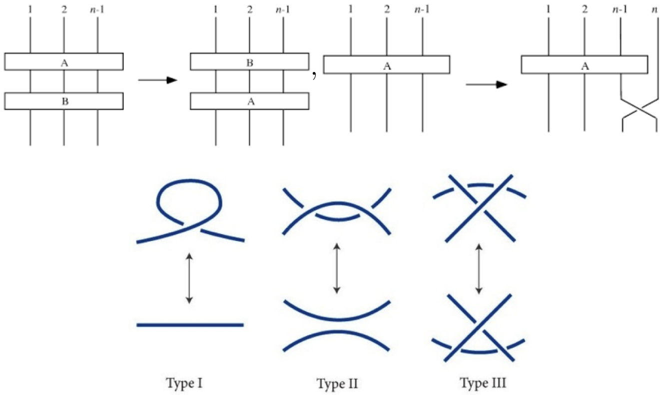

Figure : The relations: the Markov and Reidemeister moves.

(Pictures from http://mathworld.wolfram.com/MarkovMoves.html and https://www.quora.com/How-would-you-explain-knot-theory-to-a-10-year-old.)

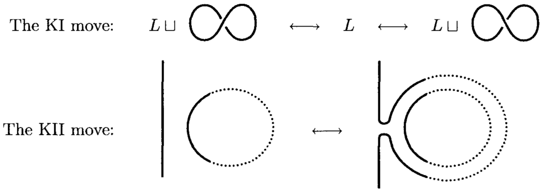

Figure : The Kirby moves.

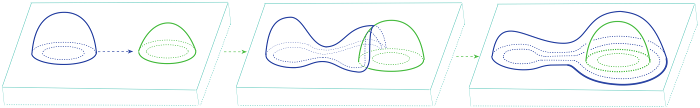

Figure : The handle slide move.

(Pictures from "Tomotada Ohtsuki, Quantum invariants" and
`http://users.math.msu.edu/users/akbulut/papers/akbulut.lec.pdf`.)

Some examples

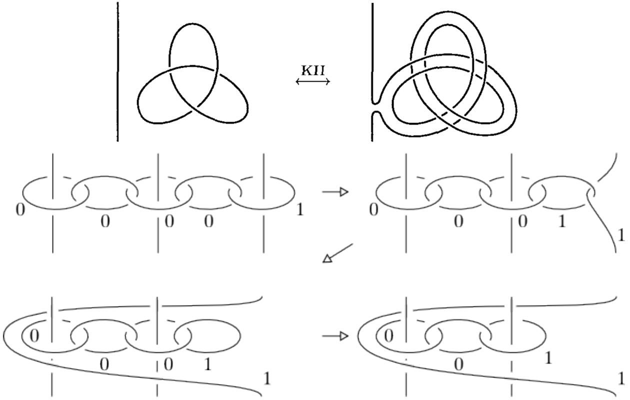

Figure : Kirby(-Fenn-Rourke) calculus in action.

(Pictures from "Tomotada Ohtsuki, Quantum invariants" and
`http://mathoverflow.net/questions/30972/kirby-calculus-and-local-moves`.)

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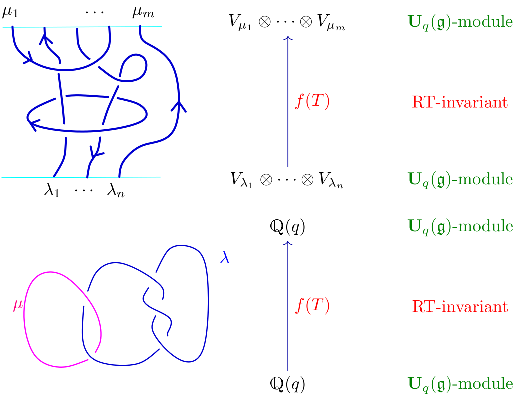

Figure : The Reshetikhin-Turaev approach.

(Picture from "Aaron Lauda, An introduction to diagrammatic algebra and categorified quantum $\mathfrak{sl}_2$".)

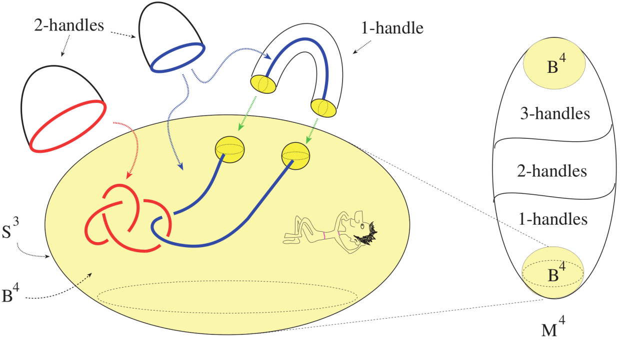

Figure : From knots/links to 4-manifolds.

(Picture from http://users.math.msu.edu/users/akbulut/papers/akbulut.lec.pdf.)

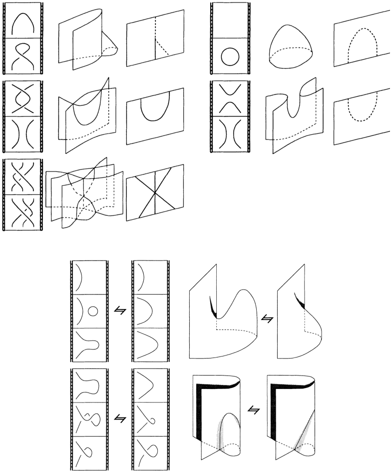

Figure : The category of links: generators and relations.

(Pictures from " Scott Carter and Masahico Saito, Knotted surfaces and their diagrams".)

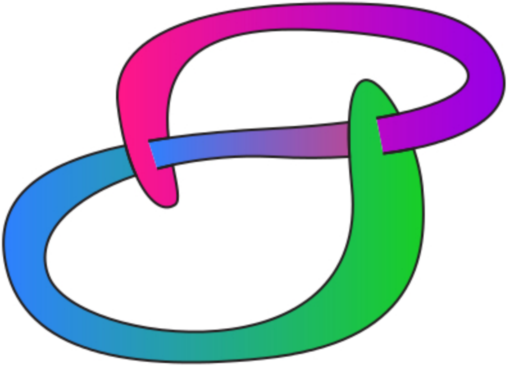

Figure : An in $\mathbb{R}^4$ embedded disk with a knot as its boundary.

(Picture from https://en.wikipedia.org/wiki/Ribbon_knot.)

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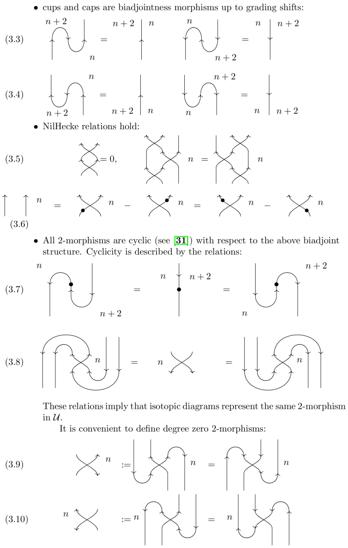


- cups and caps are biadjointness morphisms up to grading shifts:

(3.3)

(3.4)

- NilHecke relations hold:

(3.5) $= 0,$

(3.6)

- All 2-morphisms are cyclic (see [31]) with respect to the above biadjoint structure. Cyclicity is described by the relations:

(3.7)

(3.8)

These relations imply that isotopic diagrams represent the same 2-morphism in $\mathcal{U}$.

It is convenient to define degree zero 2-morphisms:

(3.9)

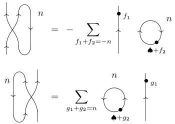
(3.10)

Figure : Relations in the categorified quantum group.

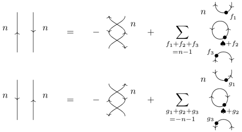
More relations

- For the following relations we employ the convention that all summations are increasing, so that $\sum_{f=0}^{\alpha}$ is zero if $\alpha < 0$.

(3.13)



(3.14)



for all $n \in \mathbb{Z}$. In equations (3.13) and (3.14) whenever the summations are nonzero they involve fake bubbles.
- the additive $\Bbbk$-linear composition functor $\mathcal{U}(n, n') \times \mathcal{U}(n', n'') \to \mathcal{U}(n, n'')$ is given on 1-morphisms of $\mathcal{U}$ by

$$\mathcal{E}_{\underline{\epsilon}'} \mathbf{1}_{n'}\{t'\} \times \mathcal{E}_{\underline{\epsilon}} \mathbf{1}_{n}\{t\} \mapsto \mathcal{E}_{\underline{\epsilon}'\underline{\epsilon}} \mathbf{1}_{n}\{t + t'\}, \tag{3.15}$$

and on 2-morphisms of $\mathcal{U}$ by juxtaposition of diagrams

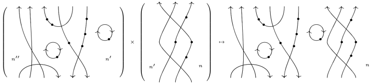


Figure : We are not done yet.

Even more

We record here some additional relations that hold in $\mathcal{U}$. See [31] for more details.

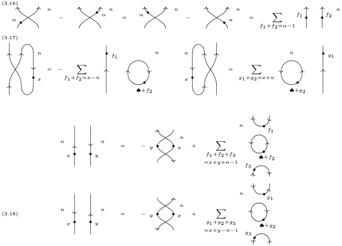


Figure : And more...

(All pictures from “Aaron Lauda, An introduction to diagrammatic algebra and categorified quantum $\mathfrak{sl}_2$”.)

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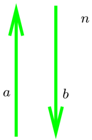
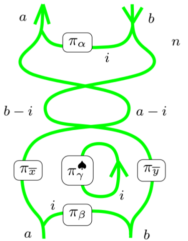
THEOREM 5.2.5 (Stošić Formula). There is an equality

(5.50)



$$= (-1)^{ab} \sum_{i=0}^{\min(a,b)} \sum_{\alpha,\beta,\gamma,x,y} (-1)^{\frac{i(i+1)}{2}+|x|+|y|} c^{K_i}_{\alpha,\beta,\gamma,x,y}$$



where the sum is over all partitions $\alpha, \beta, \gamma \in P(i)$, $x \in P(i, a-i)$, $y \in P(i, b-i)$; $K_0 = \emptyset$, and $K_i = ((n+a-b-i)^i)$ for $1 \leq i \leq \min(a,b)$.

Figure : The Stošić formula in thick calculus.

(Picture from "Mikhail Khovanov, Aaron Lauda, Marco Mackaay, Marko Stošić, Extended graphical calculus for categorified quantum $\mathfrak{sl}_2$".)

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