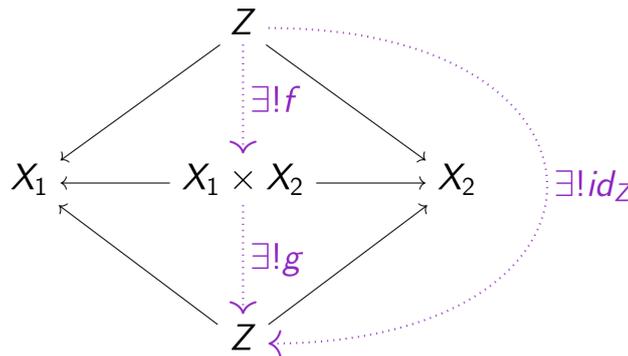


EXERCISES 7: LECTURE CATEGORY THEORY

Exercise 1. Show that products (if they exist) are unique up to unique isomorphism:



What about other limits?

Exercise 2. What are the following limits and colimits in $\mathbb{K}\text{VECT}$?

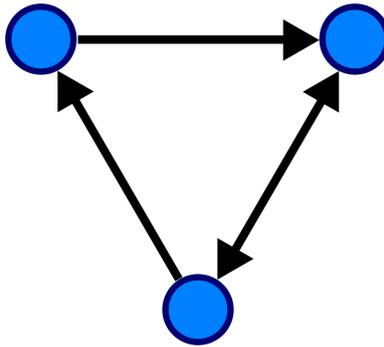
This diagram	is this functor, and	its limit is called	its colimit is called
empty	\mapsto	the terminal object	the initial object
$A \quad B \quad C$ $A \rightarrow C$	$\bullet \bullet \bullet \mapsto A \quad B \quad C$	the product	the coproduct
$A \rightarrow C$ $B \downarrow$	$\bullet \rightarrow \bullet \downarrow \bullet \mapsto A \rightarrow C$	the pullback	—
$C \rightarrow B$ $A \downarrow$	$\bullet \rightarrow \bullet \mapsto C \rightarrow B$	—	the pushout
$A_1 \leftarrow A_2 \leftarrow A_3 \leftarrow \dots$	$\bullet \leftarrow \bullet \leftarrow \bullet \leftarrow \dots \mapsto A_1 \leftarrow A_2 \leftarrow A_3 \leftarrow \dots$	the inverse limit	—
$A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \dots$	$\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \dots \mapsto A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \dots$	—	the direct limit
$A \rightrightarrows B$	$\bullet \rightrightarrows \bullet \mapsto A \rightrightarrows B$	the equalizer	the coequalizer

Exercise 3. Show that TOP is bicomplete.

Hint: Recall that a category is bicomplete if and only if it has (co)equalizers and (co)products.

Exercise 4. Let QUIVER be the category of directed graphs: objects are directed graphs, and arrows are directed graph maps, which must preserve sources and targets of edges. Decide whether

QUIVER is complete or not.



Hint: There is a forgetful functor $\text{Forget}: \text{CAT} \rightarrow \text{QUIVER}$.

- ▶ The exercises are optional and not mandatory. Still, they are highly recommend.
- ▶ There will be 12 exercise sheets, all of which have four exercises.
- ▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- ▶ The distinction between “large classes” and “small classes (sets)” turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- ▶ There might be typos on the exercise sheets, my bad, so be prepared.