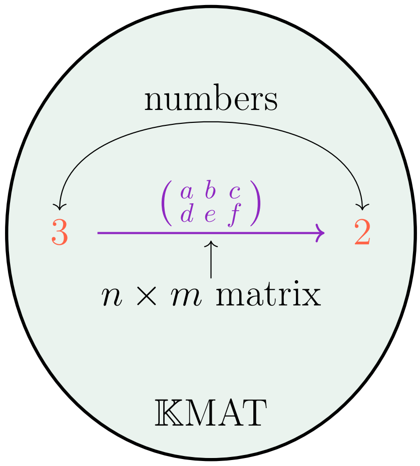
EXERCISES 4: LECTURE CATEGORY THEORY

Exercise 1. Describe the skeleton of SET.

Exercise 2. Let $\mathbb{K}$MAT be the category whose objects are $X \in \mathbb{Z}_{\geq 0}$, and whose arrows are $\mathbb{K}$-valued matrices $f: m \to n$ of size $n \times m$.

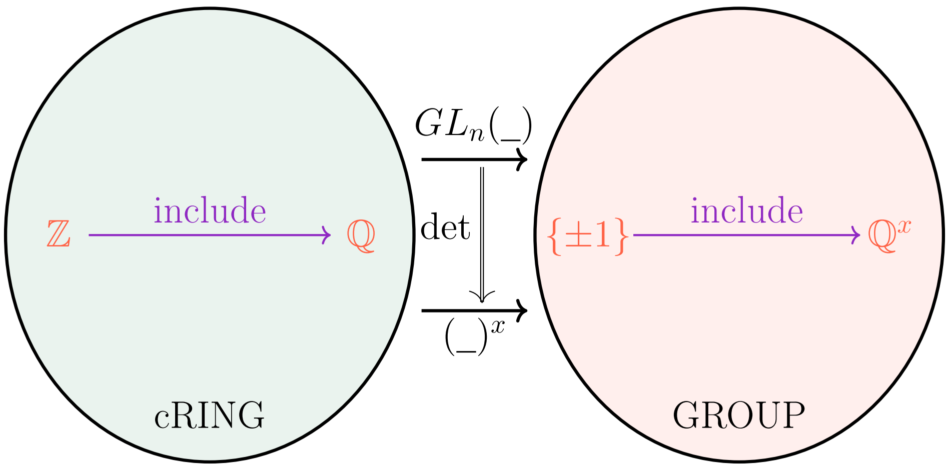
Show that $\mathbb{K}\mathrm{MAT} \simeq \mathbb{K}\mathrm{fdVECT}$, but $\mathbb{K}\mathrm{MAT} \not\cong \mathbb{K}\mathrm{fdVECT}$. (That is, show that the categories are equivalent but not isomorphic.)



Exercise 3. Show the following.

a) $GL_n(\_)$ and $(\_)^x$ (group of units) are functors from cRING to GROUP.

b) $\det: GL_n(\_) \Rightarrow (\_)^x$ is a natural transformation.



Exercise 4. Consider the categories GROUP, MONOID and SEMIGROUP. Show that

$$\mathrm{GROUP} \subset \mathrm{MONOID} \subset \mathrm{SEMIGROUP}$$

as subcategories and decide whether the two inclusions are full.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.

- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.