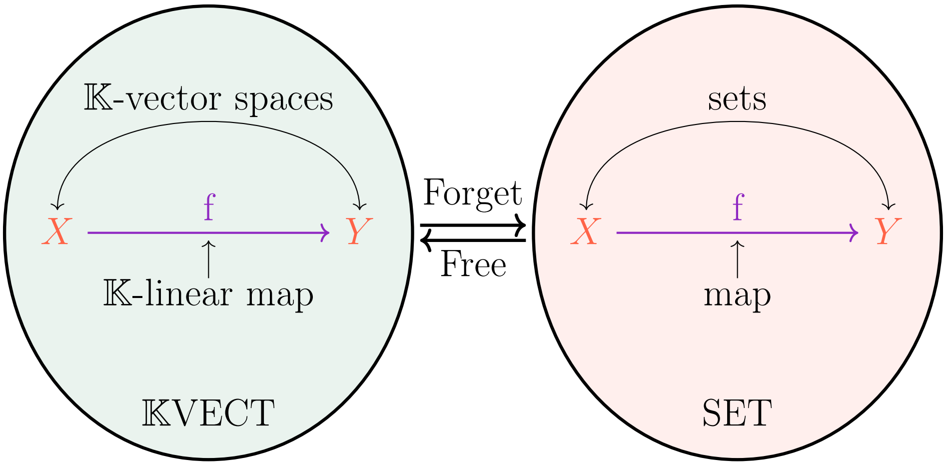
EXERCISES 3: LECTURE CATEGORY THEORY

Exercise 1. Recall that we have Forget and Free functors between $\mathbb{K}$VECT and SET:



Verify that these are indeed functors that satisfy

$$\hom_{\mathbb{K}\text{VECT}}(\text{Free}(X), Y) \cong \hom_{\text{SET}}(X, \text{Forget}(Y))$$

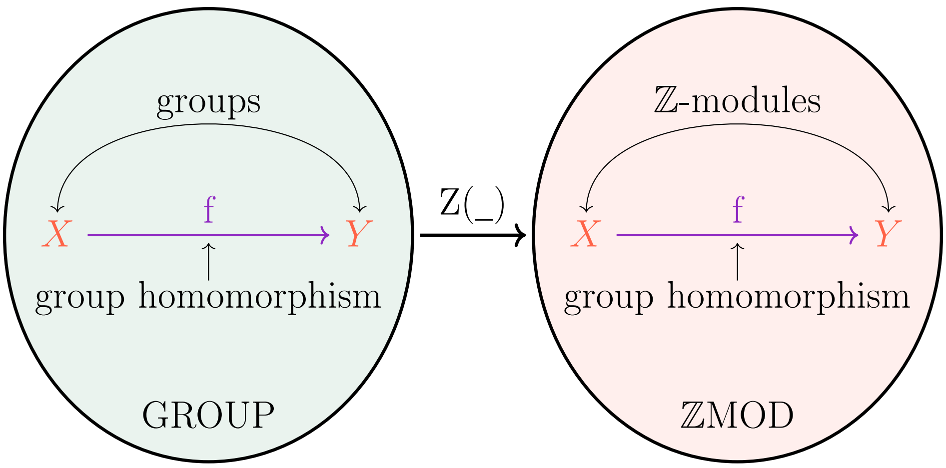
for all sets X and all $\mathbb{K}$-vector spaces Y.

Exercise 2. Let $\mathbb{Z}$MOD denote the category of $\mathbb{Z}$-modules, and let abGROUP denote the category of abelian groups. Show that $\mathbb{Z}$MOD $\cong$ abGROUP, that is, there are functors $F\colon \mathbb{Z}\text{MOD} \to \text{abGROUP}$ and $F\colon \text{abGROUP} \to \mathbb{Z}\text{MOD}$ such that

$$GF = id_{\mathbb{Z}\text{MOD}}, \quad FG = id_{\text{abGROUP}}.$$

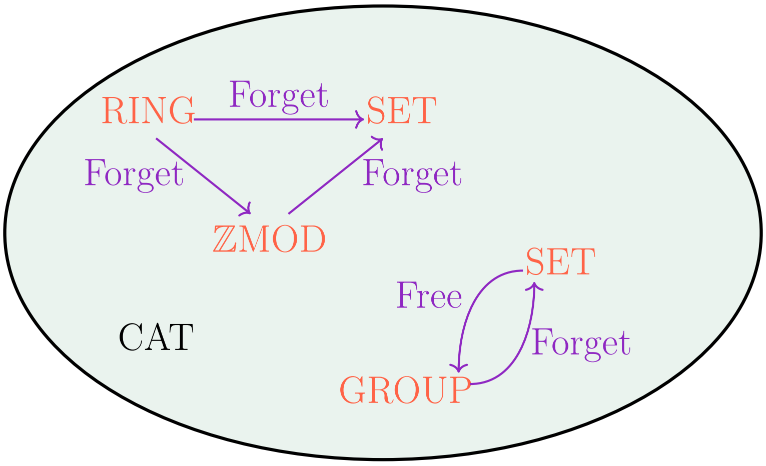
(From now on we will identify abGROUP and $\mathbb{Z}$MOD.)

Exercise 3. Decide whether there is a functor



that takes a group X to its center $Z(X)$.

Exercise 4. Verify that categories themselves form a category CAT where arrows are functors:



Are there any set-theoretical issues to be aware of?

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.