EXERCISES 1: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. Recall the following concepts from general topology, and give examples of these:

- A topology on a set and its open respectively closed sets.
- Subspace, quotient and other constructions such as products, or wedge sums.
- Continuous maps.
- Compact, connected and path-connected sets.
- Metric spaces.
- Manifold.

Check also what you recall/understand from en.wikipedia.org/wiki/General_topology

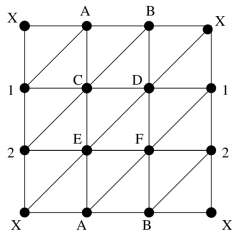
Exercise 2. The following are classical (and weird) examples in general topology:

1. The long line en.wikipedia.org/wiki/Long_line_(topology).
2. Topologists sine curve en.wikipedia.org/wiki/Topologist's_sine_curve.
3. Hawaiian earrings en.wikipedia.org/wiki/Hawaiian_earring.

Make a list what topological properties these do or do not satisfy, and argue why. (Hereby are hand-wavy arguments welcome!)

Exercise 3. Here is the Euler characteristic en.wikipedia.org/wiki/Euler_characteristic:

1. The following is a triangulation of a torus T:



   Explain why this is indeed a triangulation of a torus.
2. Use it to calculate the Euler characteristic $\chi(T) = V - E + F$, where V, E and F are the number of vertices, edges and faces in the triangulation, respectively.
3. Find a triangulation of a sphere and calculate the Euler characteristic in the same way.

Addendum:

- Hint: en.wikipedia.org/wiki/Surface_(topology)#Construction_from_polygons
- Hint: en.wikipedia.org/wiki/Tetrahedron

Exercise 4. Watch a few YouTube videos about algebraic topology such as:

- www.youtube.com/watch?v=ymF1bp-qrjU
- www.youtube.com/watch?v=AmgkSdhK4K8
- www.youtube.com/watch?v=cPg62OPdF8s (Sorry for the quality...)

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-algtop-2021.html.
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.