EXERCISES 6: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Explain in detail what the mistake in the following argument is.

Claim: Assume that $n \geq 1$ humans are in a room. Then all n humans are equal.

Proof: By induction. First, if $n = 1$, then the claim is clearly true. (Everyone is equal to him/herself.) So assume that the claim is true for some n, and assume that $n+1$ are in a room. If one human leaves the room, then all the remaining humans are equal, by induction. So let the one human reenter the room, and let another human leave the room. Again, by induction, all remaining humans are equal. Hence, all $n+1$ humans are equal. $\square$?

Exercise 2. Let X be a set containing n elements. Show that its power set $\mathfrak{P}(X)$ contains 2^n elements.

Exercise 3. Prove the following statements by induction, where $n \in \mathbb{N}_0$.

(a) $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$.

(b) $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) For $n \geq 2$ one has $n+1 < 2^n$.

(d) $n^3 - n$ is divisible by 3.

(e) $n^k - n$ is divisible by $k \in \mathbb{N}$. (Errata: Beware, this exercise was flawed. It is only true for k being a prime number, but e.g. for $k = 4$ it is possible to construct counterexamples.)

Exercise 4. Let $p \in \mathbb{N}_0, p > 1$. Show that p is prime if and only if

$$(p|ab) \Rightarrow (p|a \vee p|b) \quad \forall a, b \in \mathbb{N}_0.$$

Submission of the exercise sheet: 05.Nov.2018 before the lecture. **Return of the exercise sheet**: 08.Nov.2018 during the exercise sessions.