## EXERCISES 2: LECTURE FOUNDATIONS OF MATHEMATICS

**Exercise 1.** Let $X, Y \neq \emptyset$ be sets. Show that $(X \times Y = Y \times X) \Rightarrow X = Y$ holds.

**Exercise 2.** Let $X$ be a set, and let $\mathfrak{P}(X)$ be its power set. Show that

$$\bigcup_{A \in \mathfrak{P}(X)} A = X, \qquad \bigcap_{A \in \mathfrak{P}(X)} A = \emptyset.$$

**Exercise 3.** Let $X, Y, A, B$ be sets. Prove or disprove:

(a) $(A \times B \subset X \times Y) \Leftrightarrow (A \subset X) \wedge (B \subset Y)$ holds.

(b) $(X \times Y) \cup (A \times Y) = (X \cup A) \times Y$ holds.

(c) $(X \times Y) \cap (X \times B) = X \times (Y \cap B)$ holds.

(d) $(X \times Y) \cup (A \times B) = (X \cup A) \times (Y \cup B)$ holds.

(e) $(X \times Y) \cap (A \times B) = (X \cap A) \times (Y \cap B)$ holds.

Are some of these statements true assuming that $X, Y, A, B$ are non-empty?

**Exercise 4.** Prove the De Morgan laws: Let $\{A_i \mid i \in I\}$ be a system of sets such that $A_i \subset X$ for all $i \in I$. Then

$$\left(\bigcup_{i \in I} A_i\right)^c = \bigcap_{i \in I} (A_i)^c, \qquad \left(\bigcap_{i \in I} A_i\right)^c = \bigcup_{i \in I} (A_i)^c,$$

hold, where $^c$ denotes the complement in $X$.

**Submission of the exercise sheet**: 30.Sep.2019 before the lecture. **Return of the exercise sheet**: 03.Oct.2019 during the exercise classes.